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SIGNAL-TO-INTERFERENCE-BASED POWER CONTROL FOR WIRELESS NETWORKS: A SURVEY, 1992–2005 [1]

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Abstract. This paper summarizes and explains the main results on signal-to-interference (SIR) based power control algorithms, which are used to increase network capacity, extend battery life, and improve quality of service in cellular wireless radio systems. The classic works of Aein, Meyerhoff, and Nettleton and Alavi attracted considerable attention in the nineties. The modern approach to the power balancing control problem in wireless networks, formulated by Zander in 1992, matured in the papers of Foschini and Yates and their coworkers in the latter part of the nineties. However, the field is still wide open for research as is indicated by the increasing number of papers published in the area each year. The most recent approaches to solving the mobile power distribution problem in wireless networks use Kalman filters, dynamic estimators, and noncooperative Nash game theory.

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AMS (MOS) subject classification: 93C05, 93C95, 94A99

1 Background and Conventions

Communication networks can be fixed or mobile; however, similar power control problems are common to both types. We will not concern ourselves here with the motion of mobile stations and hence will use the terms "mobile" and "user" interchangeably.

Information travels in two directions in a wireless network: uplink (mobile-to-base) and downlink (base-to-mobile). The mathematical formulations of these problems are similar in satellite communications; they differ in cellular wireless systems. In cellular communication networks, a concern specific to the uplink is conservation of mobile battery power. Also, downlink codes are synchronous and can be made orthogonal; but uplink codes arrive at the base

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station asynchronously, resulting in cross correlation, and hence high in-cell interference potential unless power is adequately controlled. For this reason we will concentrate on the uplink.

Modern communications networks can be indoors or outdoors. It is not our purpose to delve into the differences between them. We note that the propagation mechanisms of interest to us are similar whether the network is indoors or outdoors [93].

Three types of multiple access techniques are commonly used, namely, time-, frequency- and code-division multiple access, usually known by their acronyms TDMA, FDMA, and CDMA, respectively. TDMA and FDMA protocols assign a specific time slot, respectively, frequency slot to each user. These protocols are relatively wasteful in that when a user does not transmit in its assigned slot, no other user can make use of the resource. In contrast, in a CDMA system, all users share the same time-frequency space. In CDMA, the individual signals are distinguished by encoding and decoding using distinct code sequences assigned to each user. The code bandwidth is chosen to be much larger than the signal bandwidth, generating a spread spectrum signal. Most current cellular wireless networks use CDMA or TDMA techniques.

Regardless of the access method, a common physical model is appropriate for use in power control. Specifically, we characterize the system by a gain matrix G, the meaning of whose entries depends on access method and link direction. In TDMA and FDMA, interference arises from transmissions of users assigned the same slot in nearby cells; thus a matrix entry represents effective path gain between a pair of users. In CDMA, the effective path gain g_{ij} depends on distance and code cross correlation between a pair of users in the same cell.

We address our analysis specifically to networks that use CDMA protocols because spread spectrum techniques have many advantages including:

Capacity: Combinations of powerful coding techniques and reuse of frequencies in every cell allows CDMA to provide higher capacity than TDMA.

Privacy: The code must be known in order to despread the received signal to recover the information.

Interference rejection and anti-jamming: Spread spectrum techniques are effective against both narrow band and wide band interference and jamming.

Two significant interference mechanisms are termed "near-far" and "corner" effects. The *corner effect* is observed in the downlink with the mobile approximately equidistant from three base stations (*i.e.* at a corner of a hexagonal cell). The *near-far effect* dominates in the uplink. When all mobiles transmit with equal power, the signals of mobiles close to the base station interfere strongly with those of mobiles far from the base station.

Path loss, modeled using an inverse power law relationship, is the main phenomenon that determines the values of the gain matrix entries. Spatially averaged received power, p_{rec}, at a point located a distance r from a transmitter is thus

$$p_{rec} \propto \frac{p_{trans}}{r^{\alpha}} \tag{1}$$

where p_{trans} is the transmitted power and the path loss exponent α is two in free space, somewhat higher in indoor systems[2], and generally in the range of three to five for outdoor networks [93].

Two additional phenomena that affect transmissions are shadowing and fast fading. The presence of obstructions such as buildings, hills, and trees causes slow multiplicative gain fluctuations called shadowing, which can be thought of as changing the user's effective position. Fast multipath fading results from signal reflections whose relative phases change with user motion. Assuming many multipaths and ideal Rake processing, these fluctuations can be ignored. As the effects of unmodeled shadow fading and fast fading do not change the general nature of the power control problem, we use only the inverse power law model (1).

2 History and Background

Cochannel interference resulting from frequency reuse is a major factor limiting network capacity in cellular radio systems, so a power control algorithm that reduces cochannel interference has the potential to increase network capacity. Because each user of a wireless communications system contributes to the interference affecting other users, effective and efficient power control strategies are essential for achieving both quality of service (QoS) and system capacity objectives.

The need for dynamic control of transmitted power in spread spectrum mobile communication systems was first encountered in the area of satellite communications. To fill this need, SIR (signal-to-interference ratio) balancing (also called power balancing) algorithms were proposed by Aein [1] and Meyerhoff [80] in the early 1970's. A decade later, their results were adapted by Nettleton and Alavi [87], [2], and [88] for spread spectrum mobile cellular systems. The power balancing algorithms equalize, where possible, the SIRs of all users. Although communication systems are stochastic, the power control problem leads to a purely deterministic eigenvalue problem or a linear equation, as we shall see below.

Another challenge facing communication system designers was (and still is) the mitigation of shadowing and fading effects. Bambos and Pottie [10] noted that random fluctuations due to shadowing and ducting adversely affected the worst case interference condition for TDMA systems, but had

[2]In indoor systems, the path loss exponent depends on the degree of clutter and the presence or absence of a line of sight (LoS) path.

comparatively little impact on the capacity of CDMA systems with power control. Early on, open-loop power control in wireless networks was employed to combat path loss and shadow fading [130]. Average power control techniques were proposed by Gilhousen *et al.* [29] and Viterbi *et al.* [131] to maintain the received local mean constant, thereby mitigating both near-far effects and the effect of shadowing. Ariyavisitakul [7] showed that a higher power control rate can partially accommodate the effect of fast fading.

Closed-loop power control is used in wireless communication networks to compensate for fast fading and time-varying channel characteristics, and to reduce mobile battery power consumption. The closed-loop control structure in IS-95 (one of the currently implemented CDMA standards used in wireless networks) consists of an outer loop algorithm that updates the SIR threshold every 10 ms and an inner loop which, based on SIR measurements, updates required powers at 800 Hz [123]. A block diagram illustrating the power control structure [130] is shown in Figure 1. The algorithms that we present would replace the Inner Loop Control Algorithm block. They would require additional power control bits if implemented centrally, but have the advantage that they could be implemented in a distributed manner, in which the inner loop control algorithm would be implemented at the mobile rather than the base station. In this case, the mobile would require the SIR and target SIR signals to be transmitted from the base station.

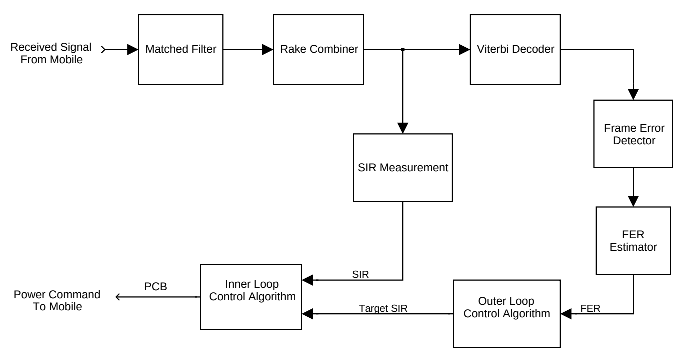

Figure 1: Power control block diagram for IS-95 CDMA standard

While power control algorithms can be classified as either open-loop or closed-loop, they can also be classified according to objective. Strength-based algorithms attempt to maintain the desired level of power. Quality of service-based algorithms attempt to maintain a performance metric such as Bit Error Rate (BER) or Signal-to-Interference Ratio (SIR) within acceptable bounds or at an acceptable value. The bit error rate generally depends on the achieved SIR. An equivalent term to SIR is Signal-to-Interference-and-Noise Ratio (SINR). A roughly equivalent term is Carrier-to-Interference Ratio (CIR). A distinction can be made between QoS-based algorithms. For

SIR-based algorithms, a crucial issue is determining the target SIR; for BER-based algorithms, the key is the derivation of a good BER estimate, given that bit errors are rare [102].

3 Power Control as an Eigenvalue/Eigenvector Problem

This section will show that if the effect of the experienced noise power of the interfering signal is sufficiently small to be neglected, formulating the power control problem mathematically leads to an eigenvalue problem involving positive matrices. Results on nonnegative matrices can be found in Gantmacher [28], Minc [81], or Varga [129].

If we let γ_d and γ_u denote the desired downlink and uplink SIRs, respectively, for all users, Nettleton and Alavi [88] showed that the corresponding balanced downlink and uplink power vectors, $\mathbf{p}_d$ and $\mathbf{p}_u$, respectively, must satisfy the eigenvalue problems

$$G\mathbf{p}_u = \frac{1+\gamma_u}{\gamma_u}\mathbf{p}_u \quad \text{and} \quad G^T\mathbf{p}_d = \frac{1+\gamma_d}{\gamma_d}\mathbf{p}_d \tag{2}$$

where the matrix G is a nonnegative matrix of known parameters whose size depends on the number of mobiles of each cell and whose entries depend on the distances from each user to each base station. (Elements of G can be thought of as effective gains.)

From (2) we can see that with $\lambda(G)$ denoting the eigenvalues of G, a solution to the SIR balancing problem, if it exists, is

$$\frac{1+\gamma_u}{\gamma_u} = \frac{1+\gamma_d}{\gamma_d} \in \lambda(G). \tag{3}$$

In the physical power control problem, the powers $\mathbf{p}_d$ and $\mathbf{p}_u$ and the SIRs γ_d and γ_u must all be positive, so (3) shows that a solution exists only if a) the matrix G has a real positive eigenvalue greater than one and b) the corresponding left and right eigenvectors are nonnegative.

We assume without loss of generality that G is irreducible, since if G is not irreducible, it should be decomposed and the subsystems analyzed separately. G then being nonnegative and irreducible, we may apply the Perron-Frobenius theorem [92] to conclude that G has a unique real eigenvalue equal to its spectral radius $\rho(G)$, whose corresponding eigenvector has all components of the same sign. The components can then all be chosen positive.

It then follows from (3) that so long as the spectral radius $\rho(G)$ satisfies $\rho(G) > 1$, one solution to the double link power balancing problem [88] is given by

$$\gamma_u = \gamma_d = \frac{1}{\rho(G)-1}. \tag{4}$$

The corresponding right and left eigenvectors of the matrix G then give the corresponding balanced powers. Of course, the eigenvectors are unique only up to a multiplicative constant, and hence would be chosen, subject to physical constraints, to minimize the power used.

The next task is to show that the spectral radius $\rho(G)$ is indeed greater than one. Consider a cellular wireless system in which n users share a channel. If the effect of background noise power is neglected, the interference $I_i(p_{-i})$ to the ith user's signal will be $\sum_{j\neq i} g_{ij}p_j$ where p_i is the transmission power corresponding to the ith user and the g_{ij} are the link gains. We use the subscript "$-i$" to indicate that the power of mobile i does not contribute to the interference its signal experiences. The SIR for the ith user is thus

$$\gamma_i = \frac{g_{ii}p_i}{I_i(p_{-i})} = \frac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j}, \quad i = 1, 2, \ldots, n. \tag{5}$$

Despite the fact that in a mobile wireless system the link gains change over time, most theoretical studies assume that the link gains to be constant.

Definition 1 *If we define the power vector by* $\mathbf{p}^T = [p_1, p_2, \ldots, p_n]$*, the SIR level* γ *is said to be* achievable *by a power control algorithm if there exists a power vector* $\mathbf{p} > 0$ *(i.e.* $p_i > 0$ *for all* i*) such that* $\gamma_i \geq \gamma$ *for all* i*.*

For convenience, we normalize the system matrix to be

$$G = \begin{bmatrix} 1 & \frac{g_{12}}{g_{11}} & \frac{g_{13}}{g_{11}} & \cdots & \frac{g_{1n}}{g_{11}} \\ \frac{g_{21}}{g_{22}} & 1 & \frac{g_{23}}{g_{22}} & \cdots & \frac{g_{2n}}{g_{22}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{g_{n1}}{g_{nn}} & \frac{g_{n2}}{g_{nn}} & \cdots & \frac{g_{nn-1}}{g_{nn}} & 1 \end{bmatrix}. \tag{6}$$

A reasonable question at this point is, "In the above described power control system, what SIR values can be achieved?". If the SIRs are required to be equal, the answer to this question was established by both Nettleton and Alavi [88] and Meyerhoff [80] and later restated by Zander [147] in a stochastic framework. The result can be stated as follows:

Lemma 1 *There exists a unique maximum achievable SIR level defined by*

$$\lambda^* = \max\{\gamma \,|\, \exists \mathbf{p} > 0 \text{ such that } \gamma_i \geq \gamma, \forall i\}. \tag{7}$$

The maximum is given by

$$\gamma^* = \frac{1}{\lambda^* - 1}, \quad \lambda^* \neq 1 \tag{8}$$

where λ^* *is the unique real positive eigenvalue of the matrix* G *for which the eigenvector problem*

$$G\mathbf{p} = \lambda^* \mathbf{p} \tag{9}$$

has a solution $\mathbf{p} > 0$. *Such* λ^* *and* $\mathbf{p}$ *exist as* G *is a nonnegative matrix. Also* $\lambda^* = \rho(G)$, *the spectral radius of* G.

If the SIRs are not constrained to be equal, the power control problem is not the standard eigenvalue problem, but leads instead to the matrix equation

$$(G - I)\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma_1} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{1}{\gamma_2} & 0 & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \ddots & 0 & \frac{1}{\gamma_{n-1}} & 0 \\ 0 & \cdots & \cdots & 0 & \frac{1}{\gamma_n} \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix}. \qquad (10)$$

We will discuss power control methods that do not require all SIRs to be equal in later chapters.

Note that since the SIR is a positive quantity, (8) indicates that, for a feasible γ^* to exist, the dominant eigenvalue of the matrix G must be not only real and positive, but also greater than one. That this does not require additional restrictions on the matrix G is easily shown using Gerschgorin's theorem, a statement of which can be found in standard linear algebra texts. Recall that the sum of the eigenvalues of a matrix is equal to the sum of the diagonal elements, which is n in the case of the normalized matrix G shown in (6). This implies that $\sum_{i=1}^{n} \lambda_i = n$, hence at least one of them (the unique real one representing the matrix spectral radius) must be greater than one.

3.1 Iterative Solution: The Power Method

In the 1970's, Meyerhoff [80] proposed finding the balanced power vector using an iterative algorithm that has the form of the *power method*, a classic method of linear algebra. Aein [1] and Meyerhoff [80] also proposed a slight modification of the power method for finding the balanced power when background (thermal) noise power is added to the interference power.

Solving the standard eigenvalue problem (9), we obtain the balanced SIR, and the corresponding powers (scaled by a constant factor). Note that solving this eigenvalue problem would require knowledge of information related to all mobiles. Accordingly, the power control would have to be implemented centrally, presumably at the base station. Disadvantages of centralized solutions are that a) it is difficult to measure the $g_{ij}, i \neq j$ and b) a solution calculated centrally must then be communicated to the users, resulting in excessive computational burden and network data flow.

On the other hand, the SIRs, which carry information about the g_{ij}'s, can be measured at any time instant. By sampling at discrete time instants

k we obtain the SIR estimates $\hat{\gamma}_i(k)$ which satisfy

$$\hat{\gamma}_i(k) = \frac{g_{ii} p_i(k)}{\sum_{j \neq i}^{n} g_{ij} p_j(k)}, \quad i = 1, 2, \ldots, n; \quad k = 0, 1, 2, \ldots \tag{11}$$

If we consider normalized power vectors $\mathbf{p}(k)$, the power method for finding the dominant eigenvalue and corresponding eigenvector iterates

$$\mathbf{p}(k+1) = \frac{1}{\|\mathbf{p}(k)\|_\infty} G\mathbf{p}(k), \quad k = 0, 1, 2, \ldots \tag{12}$$

where $\| \cdot \|_\infty$ denotes the infinity vector norm, which is equal to $\max_i \{p_i\}$. Using (6) and (11) in (12), we find that

$$p_i(k+1) = \frac{1}{\|\mathbf{p}(k)\|_\infty} \left(1 + \frac{1}{\hat{\gamma}_i(k)}\right) p_i(k), \quad i = 1, 2, \ldots, n; \quad k = 0, 1, 2, \ldots \tag{13}$$

so we see that given the measured $\gamma_i(k)$, $k = 0, 1, 2, \ldots$, and starting with almost any nonnegative initial vector $\mathbf{p}(0)$, the appropriate power can be found.

This is the "quasi-distributed" algorithm of Zander [146]. Here, "quasi" indicates that although only the individual SIR estimate is required, knowledge of the entire power vector is needed in order to find the corresponding infinity norm.

From (13) we see that the sequence of measured or estimated SIRs, if convergent, suffices to determine the balanced SIR γ^B. Assuming that the sequence of SIR measurements converges, we solve (13) for γ^B in terms of $\mathbf{p}^B$, namely

$$\gamma^B = \frac{1}{\|\mathbf{p}^B\|_\infty - 1} \tag{14}$$

where we have defined $\mathbf{p}^B$ to be the normalized power vector obtained as the limit of the right hand side of (13) as k tends to infinity.

Now that we know that the power method converges under this scenario, the next question is how fast it converges. In fact, if we order the eigenvalues of the matrix G as follows:

$$|\lambda_1| > |\lambda_2| \geq |\lambda_1| \geq \cdots \geq |\lambda_n|, \tag{15}$$

the power method is very efficient when the ratio $|\lambda_1|/|\lambda_2|$ is large; otherwise, the power method has slow convergence.

Applying the Gerschgorin theorem to the normalized matrix G in (6) tells us that all the eigenvalues of G are located in discs centered at 1 in the complex plane and that

$$|\lambda_i - 1| \leq \sum_{j \neq i}^{n} \left| \frac{g_{ij}}{g_{ii}} \right|, \quad i = 1, 2, \ldots, n. \tag{16}$$

Since in practice, it is generally the case that $g_{ii} \geq g_{ij}$, the radii of these circles are relatively small. Hence, the ratio $|\lambda_1|/|\lambda_2|$ is close to 1 so the power method is not an efficient tool for solving the power control problem of wireless systems.

In this section, we have shown that in the absence of noise, finding the balanced power control is equivalent to finding eigenvalues and eigenvectors for the system matrix. In the next section, we will show how the presence of a noise power contribution in the interference leads to a unique solution for the required power vector.

An interesting variation on the power balancing approach has been proposed recently by Wen *et al.* [136]. Their hierarchical power control algorithm first balances power among base stations, then balances SIRs among mobiles.

3.2 Power Control in the Presence of Noise: Solving a System of Linear Equations

In fact, noise is always present in a wireless network, and is not always negligible compared to the interference of the cochannel users. When the noise is explicitly included in the calculation of the interference, the signal-to-interference ratio is also sometimes called the signal-to-interference-plus-noise ratio (SINR). We now reconsider the power control problem, this time with the interference given by

$$I_i(p_{-i}) := \sum_{j \neq i} g_{ij} p_j + \sigma_i^2$$

so that the SIR of the ith mobile is

$$\gamma_i = \frac{g_{ii} p_i}{I_i(p_{-i})} = \frac{g_{ii} p_i}{\sum\limits_{j \neq i} g_{ij} p_j + \sigma_i^2}, \quad i = 1, 2, \ldots, n \tag{17}$$

where p_i is the transmission power for user i, g_{ij} is the link gain, and σ_i^2 is receiver noise (background noise) power.

Again the goal in the power control of wireless systems is that every mobile has SIR above its target value, that is

$$\gamma_i \geq \gamma_i^{tar}, \quad i = 1, 2, \ldots, n. \tag{18}$$

In the deterministic approach to the power control problem defined in (17)-(18), it is assumed that the σ_i are constant, known quantities, usually very small with respect to the interference power. Assuming equalities in (18) and knowledge of all gains g_{ij}, (17) can be represented as a system of linear algebraic equations of the form

$$A\mathbf{p} = \mathbf{b}, \quad \mathbf{p} = [p_1, p_2, \ldots, p_n]^T \tag{19}$$

with the elements of A and $\mathbf{b}$ given by $a_{ii} = 1, a_{ij} = -\gamma_i^{tar} g_{ij}/g_{ii}, j \neq i$, and $b_i = \sigma_i^2 \gamma_i^{tar}/g_{ii}$. This system can be directly solved for the p_i, using, for example, the Gaussian elimination method. However, in the currently implemented IS-95 power control mechanism, mobile i knows only its own SIR, $\gamma_i(k)$, at discrete-time instants. In contrast to the eigenvalue problem in which any multiple of the eigenvector produced a balanced solution, the addition of noise power to the interference leads to a problem which will generally have a unique solution.

The literature on numerical linear algebra contains a number of iterative methods for solving systems of linear algebraic equations that can produce the solution of (19) with the knowledge of $\gamma_i(k)$ and γ_i^{tar} only. A number of recent power control algorithms are equivalent to various numerical methods for solving linear equations. We will discuss some of these in subsequent sections.

3.3 Iterative Techniques for Mobile Power Updates

The power balancing method for solving the power control eigenvalue problem can be solved iteratively using the following algorithm, proposed by Foschini and Miljanic [25], for solving the linear equation (19)

$$\mathbf{p}(k+1) = (I - A)\mathbf{p}(k) + \mathbf{b} \tag{20}$$

where the diagonal terms a_{ii} are one and the off-diagonal terms a_{ij} are given by $a_{ij} = -\gamma_i^{tar} g_{ij}/g_{ii}$, and the elements of $\mathbf{b}$ are $b_i = \sigma_i^2 \gamma_i^{tar}/g_{ii}$. Noting that the diagonal elements of the gain matrix A are all one, we can see that (20) constitutes Jacobi iterations[3] a technique used for iterative solution of linear systems of algebraic equations.

[3]Foschini and Miljanic [25] did not derive (20) as Jacobi iterations. Instead, they assigned power evolution dynamics to cause the steady state value of the dynamic system (described by either a matrix differential or a matrix difference equation) to be the solution of the linear equation $A\mathbf{p} = \mathbf{b}$. They achieved this by modelling the mobile power dynamics using the differential, respectively, difference equation,

$$\frac{d\mathbf{p}(t)}{dt} = -\alpha\left(A\mathbf{p} - \mathbf{b}\right), \tag{21}$$

$$\mathbf{p}(k+1) = \left(I - \beta A\right)\mathbf{p}(k) + \beta\mathbf{b}, \tag{22}$$

with α being a large positive number and β chosen sufficiently small that $\beta \max_i |\lambda_i(A)| < 1$. They assumed the matrix A to be antistable, *i.e.* to have all eigenvalues in the right half plane, which is the case when A is diagonally dominant. Under this condition, the steady state solutions of (21), respectively, (22) are found to satisfy

$$\begin{aligned} 0 = -\alpha\left(A\mathbf{p}_{ss} - \mathbf{b}\right) \quad &\Rightarrow \quad A\mathbf{p}_{ss} = \mathbf{b} \\ \mathbf{p}_{ss} = \left(I - \beta A\right)\mathbf{p}_{ss} + \beta\mathbf{b} \quad &\Rightarrow \quad A\mathbf{p}_{ss} = \mathbf{b} \end{aligned} \tag{23}$$

which satisfy (19) and hence represent the required solution to the power control problem. Foschini and Miljanic derived (21) through the use of what they called a "surrogate" derivative. Note that for $\beta = 1$, (22) is equivalent to (19), and leads to (24), the distributed power control (DPC) algorithm of Grandhi *et al.* [31].

By eliminating the interference, the Jacobi iterations (20) can be reformulated as

$$p_i(k+1) = \frac{\gamma_i^{tar}(k)}{\gamma_i(k)} p_i(k), \quad i = 1, 2, \ldots, n. \tag{24}$$

This algorithm can be implemented distributively since at each time step the ith user needs only a measurement of its own SIR $\gamma_i(k)$ and power $p_i(k)$ to compute its new power command. This algorithm is called the distributed power control (DPC) algorithm [31].

Looking back at the matrix equation (20), we see that the algorithm converges only if the spectral radius (largest eigenvalue magnitude) of the matrix $I - A$ is less than one. If we use Jacobi iterations, the requirement is then that for a suitable choice of the parameter β, the spectral radius of the matrix $I - \beta A$ be less than one.

Since in a real communication system, physical limits constrain the transmit powers to be less than some maximum which we will call p_i^{max}, i.e.

$$0 \leq p_i \leq p_i^{max}, \quad i = 1, 2, \ldots, n, \tag{25}$$

algorithm (24) is then modified accordingly [34], [143] to obtain

$$p_i(k+1) = \min \left\{ p_i^{max}, \frac{\gamma_i^{tar}}{\gamma_i(k)} p_i(k) \right\}, \quad i = 1, 2, \ldots, n. \tag{26}$$

This algorithm is called the distributed constrained power control (DCPC) algorithm and its convergence properties have been studied by Yates [143].

If all gains in (6) are known, the problem of determining appropriate powers to achieve $\gamma_i \geq \gamma$ for all i can be solved by linear programming methods, as pointed out by Bock and Epstein [15] in a different context. Along the lines of the initial work of Zander, Foschini, and Yates, Zhang *et al.* [148] has developed the power control approach for wireless networks using information about intercell interference. More recently, Wu and Bertsekas [137] have presented an integer programming solution and Ramesh and Chockalingam [99] have presented a dynamic programming solution to the power control problem.

3.4 Acceleration Techniques

Convergence speed can usually be improved by a factor of two using Gauss-Seidel iterations instead of Jacobi iterations, but this may not suffice in a real-time application. Fixed-point algorithms such as (24) are usually slow to converge to the solution of the linear equation (19) because, in general, fixed-point algorithms have a linear rate of convergence.

Early attempts to speed up convergence involved incorporation of scale factors. Leung [74] proposed a formulation which can be shown to be equivalent to multiplying the identity matrix in (20) by a time-varying scale factor and setting $\mathbf{b} = 0$ (no noise). Kim *et al.* [60] proposed a similar formulation for the case in which noise is considered.

A more sophisticated approach to speeding up the convergence is to use successive over-relaxation techniques. Jäntti and Kim [52] applied this method to obtain the following second-order iterative algorithms which they called unconstrained second-order power control (USOPC):

$$\begin{aligned} p_i(k+1) &= \omega(k)\frac{\gamma_i^{tar}}{\gamma_i(k)}p_i(k) + (1-\omega(k))\, p_i(k-1), \\ & \quad i = 1, 2, \ldots, n \end{aligned} \tag{27}$$

and constrained second-order power control (CSOPC)

$$p_i(k+1) = \min\left\{p_i^{max},\ \max\left\{0,\ \tilde{p}(k)\right\}\right\}, \quad i = 1, 2, \ldots, n \tag{28}$$

where

$$\tilde{p}(k) := \omega(k)\frac{\gamma_i^{tar}}{\gamma_i(k)}p_i(k) + (1-\omega(k))\, p_i(k-1). \tag{29}$$

In (27) and (28) the relaxation parameter $\omega(k)$ has to be appropriately determined at each step of the iteration.

A novel centralized scheme that uses projection onto convex sets was proposed by Rabee *et al.* [97].

A centralized power control algorithm that uses Krylov subspace iterations, another acceleration method, was proposed by Li and Gajic in [75]. The Krylov subspace method is based on minimization at the kth iteration of the norm of the residual $r_k = b - Ax_k$ over $x_0 + \mathcal{K}_k$ where $\mathcal{K}_k$ is the kth Krylov subspace, $K_k = \text{span}\{r_0, Ar_0, \ldots, A^{k-1}r_0\}$ [58]. Conjugate gradient or General Minimum RESidual (GMRES) methods are used to solve symmetric positive definite, or unsymmetric systems, respectively. Li and Gajic tested a GMRES algorithm against the DPC and USOPC algorithms in simulation and obtained promising results.

Krylov subspace methods can also be implemented as block-wise distributed iterative processes. Such a block iterative distributed algorithm was considered in Jäntti and Kim [54] where blocks (containing any number of users) are chosen to take advantage of partially known link gain information. Their Block Power Control (BPC) algorithm is a centralized algorithm (requiring knowledge of all link gains) within a block but distributed when viewed block-wise, since no information is exchanged among the blocks. They proved convergence under appropriate conditions and showed increased convergence speed over DCPC in simulation. Specifically, they used a splitting method to iteratively solve (19). In its simplest form, this method uses matrices M and N where $A = M - N$ to obtain

$$\mathbf{p}(k+1) = M^{-1}\left(N\mathbf{p}(k) + \mathbf{b}\right) \tag{30}$$

and M and N are chosen to obtain fast convergence, subject to the constraint $\rho(M^{-1}N) < 1$ on the spectral radius.

A fully distributed approach, recently presented by Li and Gajic [76], uses Steffensen iterations to accelerate the DCPC algorithm (24). The Steffensen method can improve the convergence of an underlying fixed-point algorithm by an order of magnitude.

In still another approach, Lelic and Gajic [69] note the fact that in IS-95 power control commands are updated asynchronously. As a first step towards asynchrony, they assume synchronous updates in which, at time instant k, $m-1$ of the mobiles have already updated their powers. They then apply Gauss-Seidel iterations to accelerate the power control algorithm for each remaining mobile. Assuming without loss of generality that mobiles 1 through $m-1$ have already updated their powers at time k, they define the SIR of mobile i at time instant k by

$$\gamma_i(k, k-1) = \frac{g_{ii} p_i(k)}{\sum_{j=1}^{i-1} g_{ij} p_j(k) + \sum_{j=i+1}^{n} g_{ij} p_j(k-1) + \sigma_i^2}, \quad i = m, m+1, \ldots, n, \tag{31}$$

where all other variables are defined as before. Then applying Gauss-Seidel iterations to solve (19) leads to the update algorithm given by

$$p_i(k+1) = \frac{\gamma_i^{tar}}{\gamma_i(k+1, k)} p_i(k), \quad i = m, m+1, \ldots, n. \tag{32}$$

The equivalent constrained version of the power update algorithm is

$$p_i(k+1) = \min\left\{ p_i^{max}, \frac{\gamma_i^{tar}}{\gamma_i(k+1, k)} p_i(k) \right\}, \quad i = m, m+1, \ldots, n, \tag{33}$$

and, by introducing a relaxation parameter β in a similar manner to that of Foschini and Miljanic [25], they obtain accelerated and constrained accelerated versions of the algorithm. The accelerated version is

$$p_i(k+1) = \left[1 - \beta + \beta \left(\frac{\gamma_i^{tar}}{\gamma_i(k, k-1)} \right) \right] p_i(k), \quad i = m, m+1, \ldots, n \tag{34}$$

and the constrained accelerated version is

$$\begin{aligned} p_i(k+1) \quad &= \quad \min\left\{ p_i^{max}, \left[1 - \beta + \beta \left(\frac{\gamma_i^{tar}}{\gamma_i(k, k-1)} \right) \right] p_i(k) \right\}, \\ & \qquad i = m, m+1, \ldots, n. \end{aligned} \tag{35}$$

Their simulation results indicate that much faster convergence can be obtained using their algorithm than with algorithms (24) and (27).

Lv, Zhu, and Dong [79] propose an alternative way to accelerate convergence by setting

$$p_i(k+1) = e^{c(\gamma_i^{tar} - \gamma_i(k))} p_i(k). \tag{36}$$

Considering the Taylor series expansion of the right hand side of (36) suggests that it should be further examined whether this algorithm obtains faster convergence in one direction (increasing or decreasing power) and slower convergence in the other than the algorithms of *e.g.* Foschini and Miljanic [25].

4 Additional Literature Highlights

Having discussed the general nature of the power control problem, we now summarize various additional results found in the literature, with emphasis on SIR-based algorithms.

We mention here several other overview papers that may be of interest to the reader. The earliest by Hanly and Tse [42] in 1999 investigates network capacity issues and the efficiency of power control algorithms at minimizing power usage while meeting QoS requirements in spread-spectrum networks. In 2000, Sung and Wong [120] discussed mathematical aspects of various power control algorithms. Also in 2000, Saarinen [103] provided a very readable and thorough review of the power control literature.

4.1 Centralized and Decentralized Control Schemes

A centralized controller calculates appropriate control actions for all users based on information such as the link gains. In contrast, a distributed controller must use only locally available information to find a rational control action for a local user. Zander [146], [147] and Li and Gajic [75] proposed centralized SIR-based power control schemes, whereas Grandhi *et al.* [32], [31], [34], Zander [146], Foschini and Miljanic [25], Yates [143], and Kim *et al.* [60] proposed distributed controllers. The distributed algorithms of Zander, Grandhi *et al.*, and Foschini and Miljanic need only the user's own SIR information to calculate the local user's required power.

4.2 Effects of Processing and Propagation Delays

Most of the SIR-based power control algorithms discussed above are designed without considering measurement and processing delays. However, a number of researchers have addressed the issue of time delays in designing power control algorithms. In one approach, Gunnarsson *et al.* [40] designed PID controllers to overcome the effects of time-delay in the feedback loop. Subsequently, Gunnarsson *et al.* [41] and Gunnarsson and Gustafsson [37] [38] discussed the effects of time delays and proposed methods of compensation for SIR-based power control in CDMA systems. In particular, [37] proposed using a Smith predictor to adjust the SIR measurements to compensate for the delay. Gunnarsson's paper [35] on limitations of power control discusses the effects of delays and update rate, as well as filtering effects and measurement and feedback errors, on closed-loop power control performance.

Lee *et al.* [67] also proposed a closed-loop power control incorporating a Smith predictor to compensate the round-trip communication delay. They used a genetic algorithm to select the coefficients of a fixed-order robust H_∞ filter that minimizes the worst-case effects of interference and noise on SIR variance. Another approach to mitigating the effects of loop delay was presented in Rintamäki *et al.* [100]. They proposed several new algorithms based on minimum-variance and generalized-minimum-variance self-tuning controller structures and showed significant improvements in network capacity in simulation.

Fan and Arcak [22] derived sufficient conditions for delay robustness for a class of nonlinear interconnected systems and used their results to determine delay robustness of the game-theoretic algorithm of [4]. In [24], Fan *et al.* showed L_p stability, for $p \in [1, \infty]$, of the algorithm with respect to additive noise. They then used the L_∞ property and a loop transformation to prove global asymptotic stability for sufficiently small time-delays. By scaling down the step-size in the gradient algorithm used in the power control algorithm, they were also able to show global asymptotic stability for larger delays.

4.3 Adaptive Step Size for Power Increments

In current systems, uplink power control is achieved by sending a single bit (whose values represent the commands to increase or decrease power by one unit, 1 dB of power) to the mobile. Accordingly, some of the power control literature deals with choice of step-size and or potential improvement achievable if more than one bit is used to send the power control command.

Song *et al.* [110] gave guidelines for choosing the appropriate power control step size in IS-95. Herdtner and Chong [45] discussed convergence of algorithms in which step size is proportional to current power, then proposed a simple asynchronous distributed power control scheme based on IS-95 [123] and gave the corresponding convergence condition.

Yang and Chang [142] proposed an adaptive step-size algorithm that takes into account both signal strength and SIR through the use of adjustable thresholds. Sung and Wong [119] proposed a distributed fixed-step power control algorithm that incorporates an additional option to leave the power unchanged if the power is within an asymmetric region defined using the same scale factor as the quantization. Also, Sung *et al.* [117] proposed an adaptive threshold for fixed-step power control. Kim [59] proposed a quantized power control algorithm that could be implemented using a single power control bit by scaling the power by δ if the PCB is one and δ^{-1} otherwise and studied its convergence.

Recently, Wang and Chang [133] derived a closed-form expression for the probability of a false power control command being issued by their fixed-step size closed-loop power control algorithm. Based on their analysis, they conclude that fixed-step schemes are more robust to measurement errors than are variable-step schemes.

4.4 Shaping of Power or SIR Dynamics

In addition to the algorithms of Foschini and Miljanic [25] and Jäntti and Kim [52] mentioned above, algorithms that shape the dynamics of the controlled power or the convergence of the algorithm can be found in Berggren [12] and others. For example, Gunnarsson *et al.* [40] designed PID controllers to overcome the effects of time-delay in the feedback loop, and Uykan *et al.* [126] proposed a power control algorithm based on the PI-controller to improve convergence speed as compared to the DCPC algorithm.

Recently, Lee and Park [68] modified the algorithm of Foschini and Miljanic [25] to allow for variation in channel gain, in order to combat fast fading. Neto *et al.* [86] extended the analysis even further by also considering variation in interference and using a log scale for the SIR.

4.5 Convergence of Power Control Algorithms

Until 1995, except for certain specific applications, researchers addressed synchronous and asynchronous convergence separately. Foschini and Miljanic [25] showed that their distributed power control (DPC) algorithm converged synchronously, and Mitra [82] proved its asynchronous convergence. The convergence of a stochastic version of the DPC algorithm was considered under the assumption that the receiver (background) noise is Gaussian by Ulukus and Yates [125]. Sung and Wong [118] proposed a cooperative algorithm that assumed communication among neighboring base stations and proved its synchronous and asynchronous convergence. Sung [115] used Hölder's inequality to show that the feasible SIR-region is log-convex.

In 1995, a generalized framework for demonstrating asynchronous convergence of uplink power control algorithms was proposed by Yates [143], see also Huang and Yates [46]. The framework has since been extended by Leung *et al.* [72], Sung and Leung [116], and Luo *et al.* [77]. Altman and Altman [5] applied Yates' framework to show uniqueness and convergence results that can be obtained for the class of power control algorithms that can be formulated as noncooperative S-modular games. As it has been customary in the recent literature to use this framework to show convergence, we will provide a description of the framework here.

4.5.1 Standard interference functions

To provide a general framework for showing convergence of power update algorithms, Yates [143] introduced the concept of the *standard interference function.* A standard interference function $I(\mathbf{p})$ of the power vector $\mathbf{p} = [p_1, p_2, \ldots, p_n]$ of the n mobiles, is defined to be a function such that, for all $\mathbf{p} \geq 0$, $I(\mathbf{p})$ has the following properties:

Positivity $I(\mathbf{p}) > 0$,

Monotonicity $\mathbf{p} \geq \mathbf{p}' \implies I(\mathbf{p}) \geq I(\mathbf{p}')$,

Scalability $\forall \alpha > 1, \alpha I(\mathbf{p}) > I(\alpha \mathbf{p})$.

Further, a *standard power control algorithm* is defined to be a power control algorithm of the form $\mathbf{p}(k+1) = I(\mathbf{p}(k))$ if $I(\mathbf{p})$ is standard and feasible. By showing that such an algorithm satisfies the *synchronous convergence* and *Box* conditions required for asynchronous convergence in [14], Yates shows that such an algorithm converges to the unique fixed point $\mathbf{p}^*$. Note that feasibility (existence of the fixed point) is required.

To provide a set of basic tools for showing convergence of more complicated functions of power, Yates then showed within his framework that a number of simple functions are standard, including

$$I_j^{min}(\mathbf{p}) := \min\{I_j(\mathbf{p}), {I'}_j(\mathbf{p})\} \quad (37)$$
$$I_j^{max}(\mathbf{p}) := \max\{I_j(\mathbf{p}), {I'}_j(\mathbf{p})\} \quad (38)$$
$$I_j^{q}(\mathbf{p}) := \min\{q_j, I_j(\mathbf{p})\} \quad (39)$$
$$I_j^{\epsilon}(\mathbf{p}) := \max\{\epsilon_j, I_j(\mathbf{p})\} \quad (40)$$
$$\bar{I}_j(\mathbf{p}) := \beta \ln p_j + (1-\beta) I_j(\mathbf{p}) \quad (41)$$
$$\bar{I}_j^{dB}(\mathbf{p}) := \exp(\beta \ln p_j + (1-\beta)\ln I_j(\mathbf{p})) \quad (42)$$

where $I'(\mathbf{p})$ is another standard interference function, q is a vector of maximum allowable mobile powers, ϵ is a vector of minimum allowable mobile powers, and $\delta > 1$ and $\beta \in [0, 1)$ are constants.

4.5.2 Extension to quantized power control algorithms

Leung *et al.* [72] note that quantized power control algorithms do not satisfy the definition of a standard power control algorithm established in Yates' framework, hence a generalization is needed for dealing with real implementations. They consider *interference measures* $I(\mathbf{p})$ defined element-by-element as the ratio of the user's power to a quality of service measure that is a function of the power vector. Their interference measure is then called *standard* if it satisfies Yates' scalability and monotonicity conditions. (They note that positivity can be shown to follow from these.) They then construct a definition of a *canonical algorithm* in terms of the target region and update algorithm. Because of space considerations we will not provide further details here. They use their framework to show that their fixed-step power control algorithm [119] is canonical and hence converges.

4.5.3 Convergence of opportunistic communications algorithms

Another generalization has been proposed by Sung and Leung [116] to show convergence of what they call *opportunistic communications* algorithms. These are algorithms that take advantage of the fluctuation of the channel gain to transmit information when the signal strength is high. Such algorithms violate the monotonicity property of Yates' framework. First, they replace the monotonicity and scalability conditions with a new scalability condition. They define an interference function to be *two-sided scalable* if, for all $\alpha > 1$,

$$\left(\left(\frac{1}{\alpha}\right)\mathbf{p} \le \mathbf{p}' \le \alpha\mathbf{p}\right) \Longrightarrow \left(\left(\frac{1}{\alpha}\right) I(\mathbf{p}) \le I(\mathbf{p}') \le \alpha I(\mathbf{p})\right). \tag{43}$$

They then give several examples of two-sided scalable interference functions and present a pair of theorems that state that if a fixed point of a two-sided scalable interference function exists it is unique and the algorithm will converge synchronously to it. Finally, they extend the preceding synchronous convergence framework to consider asynchronous convergence and give corresponding conditions and theorems. They also determine sufficient conditions for *existence* of fixed points.

4.5.4 Extension to stochastic power control algorithms

Luo *et al.* [77] use recent results for stochastic approximations [16], [65] to extend Yates' framework to stochastic power control algorithms. They define a stochastic interference function $\tilde{I}_{(}\mathbf{p},\theta)$, where θ denotes the estimation noise, to be *standard* for all $\mathbf{p} \ge 0$ if it satisfies the following conditions:

Mean Condition: $E[\tilde{I}_{(}\mathbf{p},\theta)|\mathbf{p}] = I(\mathbf{p})$ where $I(\mathbf{p})$ is a standard deterministic function as defined by Yates and E is the expectation operator.

Lipschitz Condition: There exists a positive constant K_1 such that

$$\|I(\mathbf{p_1}) - I(\mathbf{p_2})\|^2 \le K_1\|\mathbf{p_1} - \mathbf{p_1}\|^2 \tag{44}$$

Growth Condition: There exists a positive constant K_2 such that

$$E\left[\|\tilde{I}(\mathbf{p},\theta) - I(\mathbf{p})\|^2|\mathbf{p}\right] \le K_2\left(1 + \|\mathbf{p}\|^2\right). \tag{45}$$

They also define a slightly weaker property, calling a stochastic interference function *quasi-standard* if for all $\mathbf{p} \ge 0$ the Lipschitz condition above is satisfied and the Mean and Growth conditions are replaced by the following conditions.

Mean Condition: $E[\tilde{I}_{(}\mathbf{p},\theta)|\mathbf{p}] = I(\mathbf{p}) + g(\mathbf{p})$ where $I(\mathbf{p})$ is a standard deterministic interference function as defined above and $g(\mathbf{p})$ is a bias which satisfies the Bias Condition below.

Bias Condition: There exists a positive constant K_3 and a sequence $1 \ge \beta(n) \ge 0$ such that

$$\|g(\mathbf{p}(n))\|^2 \le \beta(n)K_3\left(\|1 + \mathbf{p}(n)\|^2\right). \tag{46}$$

Growth Condition: There exists a positive constant K_2 such that

$$E\left[\|\tilde{I}(\mathbf{p},\theta) - I(\mathbf{p}) - g(\mathbf{p})\|^2|\mathbf{p}\right] \le K_2\left(1 + \|\mathbf{p}\|^2\right). \tag{47}$$

They note that standard stochastic interference functions are also quasi-standard but not *vice versa*. They then show that, with appropriate limits on the step-size sequence, quasi-standard converge component-wise with probability one to the smallest feasible power vector. They also discuss conditions for convergence in probability and apply their framework to several algorithms. In particular, they determined conditions for convergence of algorithms using both unbiased and biased interference estimates.

4.6 Stochastic Power Control Formulations

In recent years, researchers have used various problem formulations to address stochastic power control in wireless CDMA networks. In one of the earliest, in 1995, Mitra and Morrison [83] proposed a distributed algorithm that adapts mobile power on the basis of local measurements of mean and variance of the interference. They introduced a probabilistic QoS specification, developed an asymptotic framework for estimating orders of magnitude of quantities involved, and gave a condition for geometric rate of convergence.

A few years later, Ulukus and Yates [125] studied the stochastic power control problem formulation using matched filters and assuming white Gaussian noise. They showed that it is realistic to assume that the average of the squared matched filter output in a L-bit interval is corrupted by additive white Gaussian noise (AWGN). They assumed a very specific uplink implementation that uses binary phase shift keying (BPSK) and an L-bit averaging window, and devised a power control scheme that converges in the mean square sense to the optimal transmission power, with “optimal power” meaning the optimal power of the corresponding deterministic case.

4.6.1 Kalman Filtering Approaches

Leung [70], [71] and Leung and Wang [73] used the Kalman filter to predict interference in a TDMA system, assuming that the interference signal and its measurements are corrupted by additive white Gaussian noise (AWGN). Having obtained the predicted value for the interference, they used a simple scheme based on the defining formula for the signal to interference ratio (SIR) for mobile power updates [70], [71]. However, such a method optimizes neither mobile powers nor SIR errors, and its success is determined solely by the accuracy of the predicted interference. Shoarinejad *et al.* [109] developed dynamic channel and power allocation algorithms using the Kalman filter to provide predicted measurements for both gains and interference under the assumption that they are corrupted by AWGN. Jiang *et al.* [56] developed a technique that uses the Kalman filter for power estimation in wireless networks.

4.6.2 Estimator-Based Approaches

Qian [94] and Qian and Gajic [95] used an H-infinity filter to predict channel variation and proposed a power adjustment scheme based on SIR error optimization that theoretically converges in one iteration. In their approach, no assumption is imposed on the stochastic nature of the interference. An alternative estimator-based approach that is also independent of the specific stochastic nature of the interference is presented by Sorooshyari and Gajic in [112], [113]. They develop a control algorithm, based on prediction of channel interference, that minimizes an objective function incorporating both user-centric and network-centric metrics. Wen, *et al.* [135] proposed a linear prediction strategy to track and precompensate the variance of the short-term fading. Yoon *et al.* [145] presented an adaptive minimum-variance power control and showed, in simulation, its efficacy in conditions of shadowing and fast fading.

In a particularly versatile approach, Qian and Gajic [96] study the variance minimization problem for the weighted sum of variances of SIR error and transmission power when the SIR is corrupted by AWGN. They first propose to use additive power updates with increments proportional to the SIR error and then exploit the fact that the corresponding difference equation for the variance is represented by a linear discrete-time system driven by white noise whose variance satisfies the difference Lyapunov equation. They then devise a power controller by minimizing the corresponding components in the solution matrix of the difference Lyapunov equation. Their approach differs from other approaches used to date in that they do not assume any specific channel model. By using an estimator to estimate the channel variation, the controller can be applied in a variety of environments. Specifically, using a robust estimator they can then provide accurate estimates regardless of the statistics of channel disturbances/uncertainties.

4.6.3 Channel Model-Specific Approaches

There are also several works on stochastic power control assuming specific channel models. For example, a flat fading channel model is assumed and the evolution of the channel is described by stochastic differential equations in Charalambous *et al.* [18]. The performance index is the total transmission power. A linear programming approach is proposed; however, no iterative or distributed algorithms or simulation results are given.

Most other researchers assume Rayleigh fading. Kandukuri and Boyd [57] used a nonlinear convex optimization technique to minimize the mobile powers and the outage probability subject to some statistical assumptions on gains that include Rayleigh fading induced outage probability for every link. In Ramakrishna *et al.* [98], the SIR estimation problem was studied based on a signal subspace method using the sample covariance matrix of the received signal. In Sung and Wong [121], only a 2-user case is considered analytically where link gains are assumed to be varying with shadow fading. The multiuser case is studied only through simulation. Mori *et al.* [84] used predicted SIR measurements in a power control scheme that increased throughput. Lau and Tam [66] proposed a simple second-order approach to predicting channel gains and then compared decision rules based on five different optimization criteria to obtain lower outage probabilities than conventional strategies.

Dynamic optimization has been used to minimize power consumption by formulating power control for log-normal fading channels in a stochastic framework, see Huang *et al.* [47], [50], [48], [49], and [51].

4.6.4 Other Stochastic Power Control Algorithms

Luo and Ko [78] generated a maximum likelihood (ML) channel estimate from a combination of the common and dedicated physical channel pilot symbols. Neto *et al.* [85] proposed an algorithm that uses Taylor series to predict both path gain and interference.

Gombachika *et al.* [30] investigated predictive schemes based on both recursive-least squares (RLS) and least-mean-squares (LMS) algorithms for the Satellite Universal Mobile Telecommunication System (S-UMTS). They determined that the RLS algorithm converged faster but that the LMS algorithm exhibited superior tracking. Choi *et al.* [19] adaptively estimated channel characteristics and SIR for wideband CDMA downlink communications. They did this by exploiting correlation characteristics of the pilot signal and used their estimates to adjust the bandwidth of the channel estimation filter to improve performance.

Heikkinen and Prékora [44] introduced a stochastic programming formulation with probabilistic SIR constraints in order to state the power allocation problem as a convex optimization problem. In their formulation, the link coefficients may be assumed either normally or log-normally distributed. They present a distributed algorithm for implementing their approach.

Other researchers have proposed algorithms that incorporate receiver design. Varanasi and Das [128] studied stochastic power control for nonlinear multiuser receivers, assuming a much more complex form for the receiver than the one based on the matched filter-currently used in wireless CDMA networks.

4.7 Nonstochastic Optimization Algorithms

Some algorithms seek to solve a static optimization problem. The well known distributed constrained power control (DCPC) algorithm maximizes the minimum attained user SIR subject to maximum power constraints [33]. LQR optimal power control algorithms have been proposed and discussed by El-Osery and Abdallah [21], Koskie [61], Koskie and Gajic [62], Byun *et al.* [17], and Gajic *et al.* [27]. Sung and Wong [122] propose an optimal SIR-based control strategy for maximizing total effective rate.

In contrast, most of the algorithms discussed in the previous section solve stochastic optimization problems of one sort or another. A third type of optimization problem that has been addressed in the literature involves the choice of step size used in the inherent discretization involved in the implementation of digital cellular communication. For instance, Wu and Bertsekas [137] optimized over a set of discrete available power levels and Su and Geraniotis [114] adaptively optimized quantization of SIR error measurements.

In yet another variation, Tung and Yao [124] propose a convex programming approach to either maximize SIR subject to power constraints or minimize power consumption subject to a minimum SIR constraint. They then indicate how a 90-10 type constraint (meaning that 10 percent of the users should not use more than 90 percent of the total power) can be incorporated to improve the robustness of the convex programming approach.

4.8 Game Theoretic Approaches

An alternative framework for developing optimal power control algorithms is based on game theory [63] or economic formulations requiring the specification of a utility or cost function [4], and [55]. Various utility functions have been suggested [55], [108]. For example, Xiao, Shroff and Chong [141] use for each mobile a net utility function that is a sigmoid function of the SIR minus a linear function of power and propose a power update algorithm which they show achieves a Nash equilibrium whether or not the target SIRs are feasible. The use of pricing to promote efficiency and fairness has been discussed extensively by Saraydar *et al.* [104], [105], [106], and [107].

It has been shown in [61] that, in the case where a tradeoff between SIR error and power usage is permitted, the power control problem in wireless CDMA can be solved as a Nash game problem whose equilibrium solutions require less power than the power balancing solution. The motivation is as follows. The mobile has two conflicting objectives: on the one hand, the higher the SIR, the better the service; on the other hand, higher SIR is achieved at the costs of increased drain on the battery and higher interference to signals of other mobiles. Since some nonzero SIR level is necessary for accurate communication, it makes more sense to consider the cost of the difference between the actual SIR and the target SIR. Specifically, to explicitly incorporate the tradeoff between increased power consumption and decreasing SIR error, [61] and [63] assign to the ith mobile the cost function

$$J_i(p_i, \gamma_i) = b_i p_i + c_i(\gamma_i^{tar} - \gamma_i)^2, \tag{48}$$

where b_i and c_i are constant nonnegative weighting factors.[4] Where the power vector is $\mathbf{p} := [p_1, p_2, \ldots, p_n]^T$, the corresponding Nash equilibrium strategies are those power vectors $\mathbf{p}^*$ having the property that no individual

[4]The b_i technically have units of inverse power.

user can lower its cost by deviating from p_i^*. In other words, $\mathbf{p}^*$ satisfies

$$J_i(p_i^*, \gamma_i(\mathbf{p}^*)) \leq J_i(p_i, \gamma_i(p_1^*, p_2^*, \ldots, p_{i-1}^*, p_i, p_{i+1}^*, \ldots, p_n^*)), \quad \forall p_i, \quad \forall i = 1, 2, \ldots, n. \tag{49}$$

The resulting algorithm (obtained setting partial cost derivatives to zero to satisfy the necessary conditions for an equilibrium) is given in terms of the previous power value $p_i(k)$ and current SIR measurement $\gamma_i(k)$ by

$$p_i(k+1) = \begin{cases} \gamma_i^{tar}\left(\frac{p_i(k)}{\gamma_i(k)}\right) - \frac{b_i}{2c_i}\left(\frac{p_i(k)}{\gamma_i(k)}\right)^2 & \text{if defined, positive} \\ 0 & \text{otherwise.} \end{cases} \tag{50}$$

Note that the first term on the right hand side of (50) is equal to the right hand side of the power balancing algorithm (24). The second term is always negative for nonzero power so the Nash equilibrium powers will always be less than those generated by the power balancing algorithm. The ratio b_i/c_i, chosen by the mobile to represent its relative cost weights, determines the magnitude of the power savings. A particular advantage of this approach is that it allows mobiles, in effect, to opt out of communicating when, based on their relative weighting of costs, the cost is too high. This is in contrast to other schemes such as the truncated channel inversion scheme proposed by Ding and Lehnert [20] in which a centrally chosen criterion is used to suspend communication for mobiles whose channel is "bad".

Recently, modifications to the Newton algorithm have been presented and shown to have third-order accuracy [89], [90], [134]. Derivations and simulation results in [26] and [64] show that convergence of the fixed point Nash power control algorithm of [61] can be accelerated using Newton iterations. A brief summary of one of the methods implemented in [64] follows. The midpoint 3rd-order Newton method defined by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'((x_k + z_{k+1})/2)}, \quad k = 0, 1, 2, \ldots \tag{51}$$

where

$$z_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 0, 1, 2, \ldots \tag{52}$$

was proposed by Özban [89]. This method is less computationally expensive than the others, and is used to derive third-order algorithms for the static Nash power control as follows. Substituting for $p_i(k)$ in terms of $I_i(k)$ according to (17) in (50) yields a vector equation $\mathbf{F}(\mathbf{p}) = 0$ where

$$F_i(\mathbf{p}) = p_i - \frac{\gamma_i^{tar}}{g_{ii}} I_i(\mathbf{p}) + \frac{b_i}{2g_{ii}^2} I_i^2(\mathbf{p}), \quad i = 1, 2, \ldots n. \tag{53}$$

The partial derivatives

$$\frac{\partial F_i}{\partial p_i} = 1 \tag{54}$$

and

$$\frac{\partial F_i}{\partial p_j} = -\frac{g_{ij}}{g_{ii}} \left(\gamma_i^{tar} - \frac{b_i}{c_i g_{ii}} I_i \right), \tag{55}$$

define the derivative denoted by $\mathbf{F}'$. The midpoint third-order Newton algorithm is then given by

$$\mathbf{p}(k+1) = \mathbf{p}(k) - \left(\mathbf{F}' \left(\frac{1}{2} \left(\mathbf{p}(k) + \mathbf{q}(k+1) \right) \right) \right)^{-1} \mathbf{F} \left(\mathbf{p}(k) \right), \tag{56}$$

where

$$\mathbf{q}(k) = \mathbf{p}(k) - \left(\mathbf{F}' \left(\mathbf{p}(k) \right) \right)^{-1} \mathbf{F} \left(\mathbf{p}(k) \right). \tag{57}$$

An alternative Nash game formulation of the wireless power control has recently enjoyed considerable popularity in the research community. Alpcan *et al.* [4] proposed a formulation of the SIR-based power control problem in which each mobile uses a cost function that is linear in power and logarithmically dependent on SIR. Specifically, their ith user's cost is defined as

$$J_i(\mathbf{p}) = \lambda_i p_i - u_i \ln(1 + \gamma_i) \tag{58}$$

where λ_i and u_i are the cost weights. They establish the existence and uniqueness of the Nash equilibrium solution and consider the effect of various pricing schemes on system performance. In [4], they obtain the unique Nash equilibrium for the cost function that is the difference between their pricing and utility functions. Subsequently, in [3], Alpcan and Başar present noncooperative game theoretic power controls using a broad class of convex cost functions and prove existence and uniqueness of a Nash equilibrium for the switched hybrid system representing a multicell CDMA wireless system. They show global exponential stability of their update algorithm and investigate robustness to feedback delays and quantization.

Fan *et al.* [23] have proposed a generalization of the algorithm of [4] in which they exploit passivity properties of the gradient-type algorithm to offer design flexibility that can be used to achieve improved performance and robustness. Pavel [91] considers the related power control problem for achieving acceptable Optical Signal to Noise Ratio (OSNR) in optical networks. An appropriate utility function is introduced and existence and uniqueness of the corresponding Nash equilibrium as well as convergence of the corresponding OSNR-based power control algorithm are shown.

4.9 Other Nonlinear Algorithms

The game theoretic formulations discussed in Section 4.8 are nonlinear. However, they are not the only nonlinear algorithms that have been proposed. Song *et al.* [110], [111] and Gunnarsson *et al.* [39], [40] considered up/down power control and closed-loop power control with other nonlinear elements within the framework of nonlinear control systems. Rohwer, *et al.* [101] propose the use of a binary classification machine learning approach to determine a set of fixed power values corresponding to ranges of expected SIRs.

Uykan and Koivo [127] proposed a novel first-order fully distributed algorithm. They first develop a continuous version of the algorithm, which uses a sigmoid function and is quadratically convergent near its fixed point, then discretize to obtain an implementable algorithm incorporating upper and lower bounds on power.

4.10 Call Admission, Call Dropping, and Base Station Assignment

Bambos and Pottie [10] discussed the power control problem and its relation to the call admission problem. They also pointed out that the power control problem is time-varying. As the number of active mobiles in the cell changes over time, even the order of the problem changes.

The *outage probability* is the probability that some randomly chosen mobile has SIR below a prespecified value. The power distribution algorithm proposed in Zander [146] is "optimal" in the sense of minimizing the outage probability. More precisely, it reduces the outage probability to zero by keeping all SIRs above the given threshold, called the *system protection ratio*). Zander observes that "one somewhat surprising way to minimize the [outage] probability [is] to construct smaller and smaller balanced systems by removing cells (i.e., by turning their transmitters off)." This result should not be surprising as it is a direct consequence of the Perron-Frobenius theorem. Reducing any entry in the matrix G to zero decreases the spectral radius of the matrix, and by (8), increases the corresponding SIR.

Yates and Huang [144] proposed an integrated power control and base station assignment algorithm in which the total transmitted uplink power is minimized over power vectors and base-station assignments, subject to minimum SIR requirements.

Wu [138] shows that, whenever the minimum required SIR is not achieved, mobiles' outage probability can be dramatically decreased, and suggested dropping that mobile whose removal most improves the remaining SIRs. Along the same lines, in [139], Wu discusses heterogeneous SIR thresholds.

Distributed algorithms were considered by Andersin *et al.* [6], who proposed algorithms for call dropping in cellular PCS with DCPC and by Bambos *et al.* [9], who proposed a distributed power control algorithm with active link protection. Andersin *et al.* showed that finding the optimal removal set is an NP-complete problem. They then categorized algorithms into three general classes, which they analyzed and compared in simulation.

In recent years, the volume of literature on call admission, dropping, and base station assignment algorithms has exploded. The analyses and algorithms have become increasingly sophisticated and often include rate considerations for data transmissions. As an example of the former, we cite the paper by Ayyagari and Ephremides [8], which develops an admission power control algorithm that maintains active link quality while maximizing free capacity for new admissions. Another example is found in Xiao *et al.* [140] where a distributed power control algorithm is augmented to include admission control, providing good performance in simulation. In particular, the algorithm rejects (drops) infeasible calls, admits feasible calls and converges to the Pareto optimal power assignment. We will not further discuss rate-based algorithms as they are beyond the scope of this paper.

5 Conclusion

One of the most common approaches to closed-loop power control in wireless communication networks is SIR balancing, also called power balancing. The SIR balancing solution was originally derived for satellite communications by Aein [1] and Meyerhoff [80], and adapted for wireless communications by Nettleton [88] and Zander [146], [147]. Variations on the SIR balancing algorithm have replaced the target SIR by functions incorporating minimum allowable SIR [132], SIRs of other mobiles [43], [121], and maximum allowable power [33], [132] among others. Variations have also been developed to incorporate call admission and handoff [11], [53], [13], and base station assignment [46].

SIR balancing algorithms are simple and most can be implemented distributively, but they have the disadvantage that their convergence can be slow and is guaranteed only if every mobile's target SIR is feasible. Also, as we showed in [61] there is still considerable room for improvement in power control algorithms for wireless systems. In that document, we presented a static Nash (noncooperative) game formulation of the power control problem with simulation results indicating that substantial power savings may be achieved in exchange for small deviations in SIR error. In this overview paper, we have described several major categories of power control approaches currently being pursued in the literature.

However, as noted by Gunnarsson [35], [36], there are fundamental limits on the improvements that can be achieved through power control. He noted that the performance of any power control algorithm will be limited by update rate, feedback bandwidth, time delays, measurement errors, and filtering effects.

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