ECE569 Fall 2017 Notes: Textbook Example 4.7

This example is from the textbook by Spong *et al.*[1], which is the textbook for the ECE569 this Fall 2017 semester. I wrote the Matlab code whose output is displayed here. Any errors are mine and should be reported to me, skoskie@iupui.edu, rather than to the textbook authors.

This example discusses the textbook authors' model of the Stanford Manipulator. The Denavit-Hartenberg representation is given in Example 3.5 of the textbook and the geometric Jacobian is given in Example 4.7. The following Matlab code calculates the z_i shown in the textbook, and the corresponding Jacobian. I've suppressed the output of the results that don't fit on the page. Interested readers can use Matlab to generate them for themselves.

```
>> %%%
>> %%% Calculations for Example 4.7 of the textbook by Spong et al.
>> %%% (c) 2017 S. Koskie
>>
>> % The Ai matrices were given in Example 3.5 of the textbook...
>>
>> syms s1 s2 c1 c2
>> A1 = [c1 0 -s1 0;s1 0 c1 0;0 -1 0 0;0 0 0 1]

A1 =

[ c1,  0, -s1, 0]
[ s1,  0,  c1, 0]
[  0, -1,   0, 0]
[  0,  0,   0, 1]

>> A2 = [c2 0 s2 0;s2 0 -c2 0;0 1 0 d2;0 0 0 1]

A2 =

[ c2, 0,  s2,  0]
[ s2, 0, -c2,  0]
[  0, 1,   0, d2]
[  0, 0,   0,  1]
```

[1] Spong, M., S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, John Wiley & Sons, 2006.

```
>> syms d3
>> A3 = [1 0 0 0;0 1 0 0;0 0 1 d3;0 0 0 1]

A3 =

[ 1, 0, 0,  0]
[ 0, 1, 0,  0]
[ 0, 0, 1, d3]
[ 0, 0, 0,  1]

>> syms c4 s4
>> A4 = [c4 0 -s4 0;s4 0 c4 0;0 -1 0 0;0 0 0 1]

A4 =

[ c4,  0, -s4, 0]
[ s4,  0,  c4, 0]
[  0, -1,   0, 0]
[  0,  0,   0, 1]

>> syms c5 s5
>> A5 = [c5 0 s5 0;s5 0 -c5 0;0 -1 0 0;0 0 0 1]

A5 =

[ c5,  0,  s5, 0]
[ s5,  0, -c5, 0]
[  0, -1,   0, 0]
[  0,  0,   0, 1]

>> syms c6 s6 d6
>> A6 = [c6 -s6 0 0 ;s6 c6 0 0;0 0 1 d6;0 0 0 1]

A6 =

[ c6, -s6, 0,  0]
[ s6,  c6, 0,  0]
[  0,   0, 1, d6]
[  0,   0, 0,  1]
```

```
>> % z_i occurs in the third column of the rotation matrix component of T_0^i

>> z0 = [0 0 1]'

z0 =

     0
     0
     1

>> z1 = A1(1:3,3)

z1 =

 -s1
  c1
   0

>> T02 = A1*A2

T02 =

[ c1*c2, -s1, c1*s2, -d2*s1]
[ c2*s1,  c1, s1*s2,  c1*d2]
[   -s2,   0,    c2,      0]
[     0,   0,     0,      1]

>> z2 = T02(1:3,3)

z2 =

 c1*s2
 s1*s2
    c2

>> T03 = A1*A2*A3

T03 =

[ c1*c2, -s1, c1*s2, c1*d3*s2 - d2*s1]
[ c2*s1,  c1, s1*s2, c1*d2 + d3*s1*s2]
[   -s2,   0,    c2,            c2*d3]
[     0,   0,     0,                1]
```

```
>> z3 = T03(1:3,3)

z3 =

 c1*s2
 s1*s2
    c2

>> T04 = A1*A2*A3*A4

T04 =

[ c1*c2*c4 - s1*s4, -c1*s2, - c4*s1 - c1*c2*s4, c1*d3*s2 - d2*s1]
[ c1*s4 + c2*c4*s1, -s1*s2,   c1*c4 - c2*s1*s4, c1*d2 + d3*s1*s2]
[           -c4*s2,    -c2,              s2*s4,            c2*d3]
[                0,      0,                  0,                1]

>> z4 = T04(1:3,3)

z4 =

 - c4*s1 - c1*c2*s4
   c1*c4 - c2*s1*s4
              s2*s4

>> T05 = A1*A2*A3*A4*A5;

>> z5 = T05(1:3,3)

z5 =

 c1*c5*s2 - s5*(s1*s4 - c1*c2*c4)
 s5*(c1*s4 + c2*c4*s1) + c5*s1*s2
                 c2*c5 - c4*s2*s5

>> % the origins of frames 1 through 6

>> T01 = A1

T01 =

[ c1,  0, -s1, 0]
```

```
[ s1,  0,  c1, 0]
[  0, -1,   0, 0]
[  0,  0,   0, 1]

>> o1 = T01(1:3,4)

o1 =

 0
 0
 0

>> o2 = T02(1:3,4)

o2 =

 -d2*s1
  c1*d2
      0

>> o3 = T03(1:3,4)

o3 =

 c1*d3*s2 - d2*s1
 c1*d2 + d3*s1*s2
            c2*d3

>> o4 = T04(1:3,4)

o4 =

 c1*d3*s2 - d2*s1
 c1*d2 + d3*s1*s2
            c2*d3

>> o5 = T05(1:3,4)

o5 =

 c1*d3*s2 - d2*s1
 c1*d2 + d3*s1*s2
            c2*d3
```

```
>> o6 = T06(1:3,4)

o6 =

 c1*d3*s2 - d6*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - d2*s1
 c1*d2 + d6*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2) + d3*s1*s2
                            c2*d3 + d6*(c2*c5 - c4*s2*s5)

>> J1 = [cross(z0,(o6-o0));z0]

J1 =

 - c1*d2 - d6*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2) - d3*s1*s2
   c1*d3*s2 - d6*(s5*(s1*s4 - c1*c2*c4) - c1*c5*s2) - d2*s1
                                                          0
                                                          0
                                                          0
                                                          1

>> J2 = [cross(z1,(o6-o1));z1];

>> Joint 3 is prismatic whereas the others are revolute.
>>
>> J3 = [z2;[0 0 0]']

J3 =

 c1*s2
 s1*s2
    c2
     0
     0
     0

>> J4 = [cross(z3,(o6-o3));z3];

>> J5 = [cross(z4,(o6-o4));z4];
```

```
>> J6 = [cross(z5,(o6-o5));z5]

J6 =

                                  0
                                  0
                                  0
 c1*c5*s2 - s5*(s1*s4 - c1*c2*c4)
 s5*(c1*s4 + c2*c4*s1) + c5*s1*s2
                 c2*c5 - c4*s2*s5

>> J = [J1 J2 J3 J4 J5 J6];
```