A Nash Game Algorithm for SIR-Based Power Control in 3G Wireless CDMA Networks

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***Abstract*— We propose a new algorithm for distributed power control in cellular communication systems. We define a cost for each mobile that consists of a weighted sum of power and square of signal-to-interference ratio (SIR) error and obtain the static Nash equilibrium for the resulting costs. The algorithm requires only interference power measurements and/or SIR measurements from the base station, and converges even in cases where limits on available power render the target SIR's unattainable. Examples generated using realistic data demonstrate that in demanding environments, the Nash equilibrium power provides substantial power savings as compared to the power balancing algorithm, while reducing achieved SIR only slightly. Additional simulations show that the benefit of the Nash equilibrium power control over the power balancing solution increases as receiver noise power or number of users in the cell increases. The algorithm has the advantage that it can be implemented distributively. An additional benefit of the algorithm is that based on their chosen cost function, mobiles may choose to "opt out", *i.e.* stop transmitting, if they determine that the power required to achieve their SIR objectives is more expensive to them than not transmitting at all.**

***Index Terms*— Noncooperative games, Nash equilibrium, Power Control, Wireless Communications**

I. INTRODUCTION

BECAUSE Because each user of a wireless communications system contributes to the interference affecting other users, effective and efficient power control strategies are essential for achieving both quality of service (QoS) and system capacity objectives.

Closed-loop power control is used in wireless communication networks to compensate for fast fading, time-varying channel characteristics, and to reduce mobile battery power consumption. The closed loop control structure in IS-95 (one of the currently implemented standards used in wireless networks) consists of an outer loop algorithm that updates the SIR threshold every 10 ms and an inner loop which calculates required powers based on SIR measurements updated every 1.25ms (800 Hz) [1]. The outer loop algorithm determines the target SIR γ^{tar} based on the estimate of the frame error rate (FER). The inner loop algorithm generates a power control bit (PCB) based on the difference between the actual and target SIR's. A block diagram illustrating the power control structure [2] is shown in Figure 1.

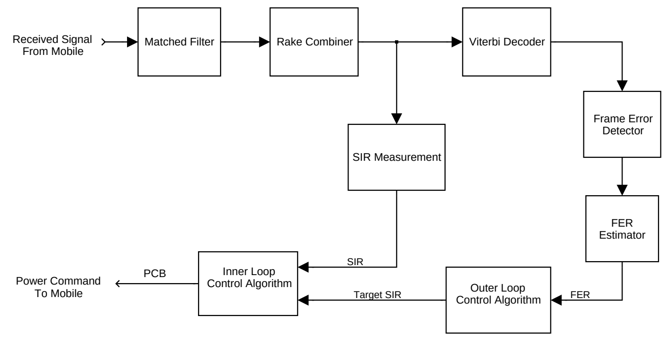

Fig. 1. Block diagram for implementation of power control in CDMA systems

A. Review of the literature

One of the most common approaches to closed-loop power control in wireless communication networks is SIR balancing, also called power balancing. The SIR balancing solution was originally derived for satellite communications by Aein [3] and Meyerhoff [4], and adapted for wireless communications by Nettleton [5] and Zander [6] and [7]. Variations on the SIR balancing algorithm have replaced the target SIR by functions incorporating minimum allowable SIR [8], SIR's of other mobiles [9] and [10], and maximum allowable power [8] and [11] among others. Variations have been developed to incorporate call admission and handoff [12], [13], and [14], base station assignment [15], and economic tradeoffs [16].

SIR balancing algorithms (SBA's) are simple and most can be implemented distributively, but have the disadvantage that convergence can be slow and is guaranteed only if every mobile's target SIR is feasible.

To address the convergence issue, a number of algorithms have been developed that shape the dynamics of the controlled power or the convergence of the algorithm [17], [18], [19], and [20]. To aid in establishing convergence of power control algorithms, a framework was proposed by Yates [21] and extended by Leung *et al.* [22].

Another class of algorithms seek to solve a static optimization problem. The well known distributed constrained power control (DCPC) algorithm maximizes the minimum attained user SIR subject to maximum power constraints [11], [21], [23]. Other algorithms minimize power consumption in the

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presence of large-scale fading [24] or over a set of discrete available power levels [25].

Dynamic optimization has been used to minimize power consumption by formulating power control for log-normal fading channels in a stochastic framework [26] and [27] as well as to adaptively optimize quantization of fedback SIR [28].

An alternative framework for developing power control algorithms is based on game theory or economic formulations requiring the specification of a utility or cost function [29], [30] and [31]. Various utility functions have been suggested [30], [32], and [16]. The use of pricing to promote efficiency and fairness has been discussed extensively [33], [34], [35], and [36]. Alpcan *et al.* [29] recently proposed a Nash game formulation of the SIR-based power control problem in which each mobile uses a cost function that is linear in power and logarithmically dependent on SIR. They establish the existence and uniqueness of the Nash equilibrium solution and consider the effect of various pricing schemes on system performance.

B. Interference Model

Power control for either the uplink (reverse link) or the downlink (forward link) can be considered. In the former case, a desirable property for a power control algorithm is the sufficiency of measurements available at the mobile for computing the power updates. Such algorithms can be implemented without reliance on communication with either the base station or other mobiles and hence are called distributed. Note that it has been shown that the same problem formulation can be applied to various types of both uplink and downlink scenarios so our discussion here is not exclusively applicable to uplink power control.

The goal in the power control of wireless systems is to ensure that no mobile's SIR γ_i falls below its threshold γ_i^{tar} chosen to ensure adequate QoS, *i.e.* to maintain

$$\gamma_i \geq \gamma_i^{tar}, \quad \forall i, \tag{1}$$

where the subscript i indexes the set of mobiles. In IS-95, this threshold is calculated for the individual mobile to maintain a satisfactory frame-error rate (FER). From the mobile's perspective, however, whether the other users meet their QoS requirements is irrelevant. For this reason, the framework of noncooperative game theory [37] is well suited for analyzing and solving the power control problem.

Considering the uplink for a single cell CDMA system with N users, we designate the transmitted power and SIR for the ith user by p_i and γ_i, respectively. We denote the background (receiver) noise power within the user's bandwidth by η_i. In the deterministic formulation of the power control problem for wireless networks, the noise power η_i is treated as constant. We use a "snapshot" model, assuming that link gains evolve slowly with respect to the SIR evolution. In this problem formulation, the SIR of the ith mobile is

$$\gamma_i = \frac{h_i p_i}{\sum_{j \neq i} h_j p_j c_{ij} + \eta_i} \tag{2}$$

where h_i is the attenuation from the ith mobile to the base station and c_{ij} is the code correlation coefficient. The attenuation is calculated from the distance r_i between the mobile and base station to be $h_i = A/r_i^{\alpha}$ in the absence of shadow and fast fading. A is a constant gain and α is usually between 3 and 6. We will provide realistic values for these constants in the simulation section, Section III. The code correlation coefficient c_{ij} is computed from the signatures $\mathbf{s_i}$ and $\mathbf{s_j}$ to be $c_{ij} = (\mathbf{s_j}^T \mathbf{s_i})^2$.

We note that this model is consistent with the general power control problem for wireless communication systems in which the SIR of mobile i is given by

$$\gamma_i = \frac{g_{ii} p_i}{I_i(\mathbf{p_{-i}})} = \frac{g_{ii} p_i}{\sum_{j \neq i} g_{ij} p_j + \eta_i} \tag{3}$$

with the interference given by

$$I_i(\mathbf{p_{-i}}) := \sum_{j \neq i} g_{ij} p_j + \eta_i. \tag{4}$$

We have used the subscript "$-i$" to indicate that the interference depends on the powers of all users except the ith. If we define a power vector $\mathbf{p}$ having ith element p_i, and an interference vector $\mathbf{I}$ having ith element $I_i(\mathbf{p_{-i}})$, the subscript indicates that the ith element of the interference vector depends on all but the ith element of the power vector.

Comparing (2) and (3), we see that for CDMA uplink power control,

$$g_{ij} := \begin{cases} h_i & j = i \\ h_j (\mathbf{s_j}^T \mathbf{s_i})^2 & \text{otherwise} \end{cases} \tag{5}$$

so g_{ij} denotes an effective link gain from the jth user to the base station that specifies the jth user's contribution to the interference affecting the signal of the ith user. We will also define an effective gain matrix $\mathbf{G}$ having (i, j)th element g_{ij}. Note that in contrast to the case in which background noise power is neglected and the diagonal elements of the gain matrix are set to zero, we cannot write the interference as the product of the gain matrix and power vector, *i.e.* $\mathbf{I} \neq \mathbf{G}\,\mathbf{p}$.

C. Motivation

The standard in the literature to which other SIR-based algorithms are compared is the DCPC algorithm for solving the SIR balancing problem for wireless communication networks [11], [21], and [23]. Although various types of optimal solutions have been considered, most of these require prohibitively large computational resources. Approaches that have been proposed include formulating the minimization of total power usage given quantized power levels as an integer programming problem [25], and minimizing outage probability as a linear programming problem [38]. More recently, it has been shown that minimizing power usage subject to power constraints or *vice versa* can be posed as a geometric programming or nonlinear convex optimization problem [39].

The first criterion by which a practical optimal power control algorithm must be judged is demonstration of significant performance improvement. Our simulation results presented below establish that *significant decreases in mobile powers can be achieved with minor effects on SIR*. Accordingly, there is a need for power update algorithms that take into account power optimization. The second criterion is robustness to environmental changes, namely receiver noise, to reduce outage.

To motivate the need for alternatives to SIR balancing algorithms we provide the following simple example.

Motivating Example: Consider a wireless system with three users whose gains are given by

$$G = \begin{bmatrix} 1.0000 & 0.0882 & 0.0357 \\ 0.1524 & 0.9500 & 0.3501 \\ 0.0767 & 0.0244 & 0.9900 \end{bmatrix}.$$

Let the receiver noise power (in appropriate units) be $\eta_1 = \eta_2 = \eta_3 = \eta = 0.01$ and assume that we want to achieve $\gamma_1^{tar} = \gamma_2^{tar} = \gamma_3^{tar} = \gamma^{tar} = 5$ via power balancing. Note that this quality of service (the desired SIR target value) corresponds to 7 dB.

By running the power balancing algorithm for 400 iterations we have obtained the following results

$$\begin{aligned} \mathbf{p} &= [6.131, 12.204, 3.928] \\ \gamma &= [4.998, 4.998, 4.998]. \end{aligned}$$

In Table I, we present the results for three mobiles and investigate the powers required to achieve equal and distinct target SIR's. The column headings indicate the specified target SIR's (in dB) for the three mobiles. The rows indicate the mobile powers required to achieve these target SIR's. The total mobile power (sum of the powers in the second column) required to achieve equal SIR's is approximately three times that required if we relax the SIR requirements only slightly (third column). The next column of the table shows that if we relax the SIR requirements further, we can again reduce the total required power by two-thirds. This example suggests that significant reductions in mobile power may be achieved if we consider alternatives to SIR balancing algorithms, especially those that minimize mobiles' power while allowing reasonable deviations from the target SIR.

II. Nash Game Problem Formulation

In the following subsections we formulate the SIR-based power control problem as a noncooperative game, choose an appropriate cost function, and find the corresponding Nash equilibrium [37], [40] power vector. We then design a power control algorithm that uses only measured information available to the individual mobile, hence can be implemented distributively. We derive conditions under which we can show convergence of the algorithm within the framework of Yates [21] and point out useful limiting behavior.

A. Cost Function and Derivation of the Nash Equilibrium

We associate with the ith user the cost function $J_i(p_i, \gamma_i(\mathbf{p}))$ where the power vector is $\mathbf{p} := [p_1, p_2, \ldots, p_N]^T$. The corresponding Nash equilibrium strategies are those power vectors $\mathbf{p}^*$ having the property that no individual user can lower its cost by deviating from p_i^*. In other words, $\mathbf{p}^*$ satisfies

$$J_i(p_i^*, \gamma_i(\mathbf{p}^*)) \le J_i(p_i, \gamma_i(p_1^*, p_2^*, \ldots, p_{i-1}^*, p_i, p_{i+1}^*, \ldots, p_N^*)), \quad \forall p_i, \quad \forall i = 1, 2, \ldots, N. \quad (6)$$

The mobile has two conflicting objectives. On the one hand, the higher the SIR, the better the service. On the other hand, higher SIR is achieved at the costs of increased drain on the battery and higher interference to signals of other mobiles. Accordingly, we define a cost function for each user depending on power and SIR. Since some nonzero SIR level is necessary for accurate communication, we consider the cost of the difference between the actual SIR and the target SIR that is chosen based on the estimated frame error rate.

A cost function should be convex and nonnegative to allow existence of a nonnegative minimum. Power is always positive in this application; however, the SIR error may be either positive or negative. To ensure positivity and convexity of the cost function, we square the SIR error term. We thus consider the following candidate cost function[1]

$$J_i(p_i, \gamma_i) = b_i p_i + c_i(\gamma_i^{tar} - \gamma_i)^2, \quad (7)$$

where b_i and c_i are constant nonnegative weighting factors.[2] We will show below that (7) provides existence of a meaningful Nash equilibrium solution.

Since the cost function depends on parameters b_i and c_i, the results obtained will depend on b_i and c_i. It is not difficult to derive the corresponding sensitivity functions of J_i with respect to b_i and c_i. These are given by

$$S_{b_i} := \frac{\partial b_i / b_i}{\partial J_i / J_i} = 1 + c_i(\gamma_i^{tar} - \gamma_i)^2 = 1 + c_i \Delta_i^2 \approx 1$$

since at the Nash equilibrium the deviation of the actual SIR from the target SIR is small, and

$$S_{c_i} := \frac{\partial c_i / c_i}{\partial J_i / J_i} = 1 + b_i p_i^*,$$

respectively. Hence, the chosen cost is more sensitive to variations in the SIR error than to variations in power, unless b_i is taken as very small.

We will see in the analysis below that only the ratio of the power weight b_i to the SIR-error weight c_i is important; hence, the c_i can be made equal to one by replacing b_i by $\tilde{b}_i = b_i / c_i$. For different applications, different ratios b_i/c_i may be chosen. Choosing $b_i/c_i > 1$ places more emphasis on power usage whereas $b_i/c_i < 1$ places more emphasis on SIR error.

Applying the necessary conditions for a Nash equilibrium we have

$$\begin{aligned} \frac{\partial J_i}{\partial p_i} = 0 &= b_i - 2c_i(\gamma_i^{tar} - \gamma_i)\frac{\partial \gamma_i}{\partial p_i} \\ &= b_i - 2c_i(\gamma_i^{tar} - \gamma_i)\frac{g_{ii}}{\sum_{j \ne i} g_{ij} p_j + \eta_i}. \end{aligned} \quad (8)$$

Recalling that $I_i(\mathbf{p_{-i}}) := \sum_{j \ne i} g_{ij} p_j + \eta_i$, and rearranging terms yields

$$\gamma_i = \gamma_i^{tar} - \frac{b_i I_i(\mathbf{p_{-i}})}{2 c_i g_{ii}}. \quad (9)$$

It follows from (9) that as $b_i \to 0$ (power expenditure ceases to be important) that $\gamma_i \to \gamma_i^{tar}$. On the other hand, as

[1] We have experimented with several choices for the cost function and found this to be both flexible and useful.

[2] The b_i technically have units of inverse power.

TABLE I
BALANCED SOLUTION POWER REQUIRED TO ACHIEVE SPECIFIED TARGET SIR VECTOR

$(\gamma_1, \gamma_2, \gamma_3)^\dagger$	(7.00,7.00,7.00)	(7.00,6.90,6.90)	(7.00,6.75,6.75)	(7.00,6.75,6.50)	(7.00,6.50,6.50)
p_1	6.432	2.308	1.090	0.814	0.592
p_2	12.804	4.523	2.078	1.532	1.080
p_3	4.120	1.472	0.689	0.496	0.369

† SIR's are measured in dB. Units of power are mW.

$c_i \to 0$, (only power usage matters and SIR value of negligible importance), γ_i no longer converges to γ_i^{tar}.

Substituting for γ_i from (3) and isolating p_i, we can express the required power in terms of given and measured quantities as

$$p_i = \frac{\gamma_i^{tar}}{g_{ii}} I_i(\mathbf{p_{-i}}) - \frac{b_i I_i^2(\mathbf{p_{-i}})}{2c_i g_{ii}^2}. \quad (10)$$

Substituting for the interference using (3) in (9), and evaluating at the Nash equilibrium we have

$$\gamma_i^* = \begin{cases} \gamma_i^{tar} - \frac{b_i}{2c_i g_{ii}} \left(\frac{g_{ii} p_i^*}{\gamma_i^*} \right) & \text{if this quantity is nonnegative} \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

Of course the equilibrium power corresponding to zero SIR is $p_i^* = 0$. Otherwise (11) yields an expression for the Nash equilibrium power p_i^* in terms of the cost weighting coefficients, the target SIR, and the Nash equilibrium SIR γ_i^*, namely

$$p_i^* = \frac{2c_i}{b_i} \gamma_i^* (\gamma_i^{tar} - \gamma_i^*). \quad (12)$$

As expected, the Nash (noncooperative) equilibrium has SIR γ_i^* less than γ_i^{tar}. When mobiles cooperate, as they must in the application of the power balancing algorithm, the target SIR, *if feasible*, will be attained by all mobiles.

Our first constraint on power arises from the fact that (10) is a quadratic equation in $I_i(\mathbf{p_{-i}})$. For given values of g_{ii}, γ_i^{tar}, and p_i, (10) has a real solution $I_i(\mathbf{p_{-i}})$, if and only if

$$p_i \leq \frac{c_i \left(\gamma_i^{tar}\right)^2}{2b_i}. \quad (13)$$

We see that if we wish to use the entire range of powers $0 \leq p_i \leq p_i^{max}$, we must choose b_i and c_i to satisfy

$$\frac{b_i}{c_i} \leq \frac{(\gamma_i^{tar})^2}{2p_i^{max}}. \quad (14)$$

For $p_i^{max} = 600$ mW and $\gamma_i^{tar} = 5$, this yields the constraint $b_i/c_i \leq 1/48$.

It is interesting to note that the power balancing algorithm is a special case of the static Nash algorithm corresponding to the cost $J_i^{PB} = c_i(\gamma_i^{tar} - \gamma)^2$ which depends only on SIR error.

B. Algorithm for Power Updates

In this section we present a numerical algorithm for solving (10). We assume that the algorithm will run in real time with measurements potentially updated every step of the algorithm. We propose the following algorithm in terms of the measured interference (an easily measured quantity)

$$p_i^{(k+1)} = \begin{cases} \frac{\gamma_i^{tar}}{g_{ii}} I_i^{(k)} - \frac{b_i}{2c_i} \left(\frac{I_i^{(k)}}{g_{ii}} \right)^2 & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where $p_i^{(k)}$ is the power of the ith mobile and $I_i^{(k)}$ the measured interference experienced by the ith mobile at the kth step of the algorithm. Recall that $I_i^{(k)} = \sum_{j \neq i} g_{ij} p_j^{(k)} + \eta_i$.

In implementation, of course, power cannot become negative so there is an implicit assumption that whenever this expression is negative, the assigned power will be zero.

In order to analyze its convergence, we rewrite the power update algorithm in the form $p_i^{(k+1)} = f_i^{(k)}(p_i^{(k)})$ in terms of the previous power value $p_i^{(k)}$ and current SIR measurement $\gamma_i^{(k)}$ as

$$f_i(p_i^{(k)}) := p_i^{(k+1)} = \begin{cases} \gamma_i^{tar} \left(\frac{p_i^{(k)}}{\gamma_i^{(k)}} \right) - \frac{b_i}{2c_i} \left(\frac{p_i^{(k)}}{\gamma_i^{(k)}} \right)^2 & \text{if defined, positive} \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

where we have substituted for $I_i^{(k)}$ in (15) using $\gamma_i^{(k)} = g_{ii} p_i^{(k)} / I_i^{(k)}$. The initial condition associated with (16) must satisfy $p_i^{(0)} \neq 0$. Note that the positive term in the expression in (16) for $p_i^{(k+1)}$ is identical to the power balancing solution (36) and the negative term is proportional to the square of the interference (compare (15)). Since, in general, the interference is small, it's square is even smaller and there is little danger that the algorithm will generate a negative power. Again, the power is, of course, also required to be nonnegative.

As noted above algorithm (16) differs from the power balancing algorithm (34) in that, to the linear (in power) term, a quadratic (in power) term is added. Since the proposed algorithm is a nonlinear algorithm it has in general (much) faster convergence than the corresponding linear algorithm. In a recent conference paper [41], we have shown how to use the Newton iterations to accelerate convergence for the power updates proposed in (16) and obtain the quadratic rate of convergence.

The two formulations of the algorithm have in common that they require only a single measurement at each step, hence if this measurement is made available to the mobile, the power algorithm can be used to implement a distributed power control. One minor difference between the two formulations of the algorithm is that the formulation in terms of interference (15) and the formulation in terms of power (16) is that the formulation in terms of power, like the power balancing algorithm, cannot be initialized with zero power. The formulation in terms of interference, however, does not require an initial nonzero power as the interference, which includes the noise power, is never zero.

C. Convergence

Yates [21] showed that if a fixed point of the algorithm $p^{(k+1)} = f(p^{(k)})$ exists and if the function f satisfies three properties: positivity $f(p) > 0$, monotonicity $p > p' \implies f(p) > f(p')$, and scalability $f(\alpha p) < \alpha f(p) \quad \forall \alpha > 1$, then the algorithm converges to the fixed point, which is unique. We show below that positivity and monotonicity of f impose constraints on acceptable values of I_i but scalability restricts the allowable receiver noise power level, and generates a limit weaker than that required for monotonicity. For readability, we will drop the index (k) in the calculations below.

From (15) we see that, in terms of the observed interference, positivity requires

$$I_i < \frac{2c_i g_{ii}\gamma_i^{tar}}{b_i}, \quad \forall i \in \{1,2,\ldots,N\}. \tag{17}$$

To simplify the equations, where necessary, in the following derivations we define

$$q_i := \sum_{j\neq i} g_{ij}p_j \tag{18}$$

and

$$q_i' := \sum_{j\neq i} g_{ij}p_j'. \tag{19}$$

To determine conditions that ensure monotonicity we then write, using (4),

$$\begin{aligned} f_i(p) - f_i(p') = {} & \frac{\gamma_i^{tar}}{g_{ii}}\left(q_i - q_i'\right) - \\ & \frac{b_i}{2c_i g_{ii}^2}\left[q_i^2 - (q_i')^2 + 2\eta_i\left(q_i - q_i'\right)\right] \\ = {} & \left(\frac{\gamma_i^{tar}}{g_{ii}} - \frac{b_i\eta_i}{c_i g_{ii}^2}\right)\left(q_i - q_i'\right) - \\ & \frac{b_i}{2c_i g_{ii}^2}\left(q_i + q_i'\right)\left(q_i - q_i'\right). \end{aligned} \tag{20}$$

Accordingly we need

$$\frac{c_i\gamma_i^{tar}g_{ii}}{b_i} \geq \eta_i + \frac{1}{2}\sum_{j\neq i} g_{ij}\left(p_j + p_j'\right) \tag{21}$$

or, noting that $p_j \geq p_j' \ \forall j \implies I_i(p) \geq I_i(p')$ we see that a sufficient condition for monotonicity is

$$I_i(p) \leq \frac{c_i\gamma_i^{tar}g_{ii}}{b_i}, \quad \forall i \in \{1,2,\ldots,N\}. \tag{22}$$

for all mobiles, which is stronger than the condition (17) needed for positivity.

Condition (17) and its stronger variant (22) determine the upper bound for the interference that guarantees the algorithm's convergence. Note that this upper bound is proportional to the product $g_{ii}\gamma_i^{tar}$. The proportionality factor is the ratio of the design parameters c_i/b_i. Note that in Section II-A, we have demonstrated using realistic data that in order to use the full mobile power range $0 \leq p_i \leq p_i^{max} = 600$ mW when $\gamma_i^{tar} = 5$, the ratio c_i/b_i should satisfy $c_i/b_i \geq 48$. Hence, the upper bound for the interference as defined by (22) is relatively large. There is no need to lower the value for b_i and increase this already large upper bound for the interference. By reducing b_i we put less emphasis on power optimization, and in the extreme case when $b_i = 0$ (which leads to the power balancing algorithm), *the power is not optimized at all*.

To determine any conditions required to preserve scalability, we again use (15), obtaining

$$\begin{aligned} \alpha f_i(p) - f_i(\alpha p) \;=\; & \frac{\gamma_i^{tar}}{g_{ii}}(\alpha - 1)\eta_i - \\ & \frac{b_i}{2c_i g_{ii}^2}\left[(\alpha - \alpha^2)q_i^2 + (\alpha - 1)\,\eta_i^2\right] \\ =\; & \frac{(\alpha-1)b_i}{c_i g_{ii}}\left[\frac{c_i\gamma_i^{tar}\eta_i}{b_i} - \frac{\eta_i^2}{2g_{ii}} + \frac{\alpha}{2g_{ii}}q_i^2\right]. \end{aligned} \tag{23}$$

Noting that

$$\left(\sum_{j\neq i} g_{ij}p_j\right)^2 = I_i^2 - 2\eta_i\left(\sum_{j\neq i} g_{ij}p_j\right) - \eta_i^2 \tag{24}$$

we make the substitution in (23) to obtain

$$\begin{aligned} \alpha f_i(p) - f_i(\alpha p) \;=\; & \frac{(\alpha-1)b_i}{c_i g_{ii}}\left[\frac{c_i\gamma_i^{tar}\eta_i}{b_i} - \frac{\eta_i^2}{2g_{ii}} + \right. \\ & \left. \frac{\alpha}{2g_{ii}}\left(I_i^2 - 2\eta_i q_i - \eta_i^2\right)\right]. \end{aligned} \tag{25}$$

The first factor on the right hand side is always positive for $\alpha > 1$ so we need only consider the second factor. Since the scalability condition must hold for all $\alpha > 1$, the condition for scalability reduces to

$$\frac{2c_i g_{ii}\gamma_i^{tar}\eta_i}{b_i} + I_i^2 > 2\eta_i\left(\sum_{j\neq i} g_{ij}p_j\right) + 2\eta_i^2 = 2\eta_i I_i \tag{26}$$

which is equivalent to

$$\left(I_i - \eta_i\right)^2 + \frac{2c_i g_{ii}\gamma_i^{tar}\eta_i}{b_i} - \eta_i^2 > 0 \tag{27}$$

or since $\eta_i > 0$,

$$\eta_i < \frac{2c_i g_{ii}\gamma_i^{tar}}{b_i}, \quad \forall i \in \{1,2,\ldots,N\} \tag{28}$$

is a sufficient condition for scalability. Note that *the scalability condition does not restrict the allowable interference but rather the allowable noise power*. However, since $\eta_i < I_i$ by definition, this sufficient condition is weaker than the earlier condition for (17) positivity.

In fact, since η_i is necessarily at most I_i, we see that the condition (28) is weaker than the condition (22) previously derived for monotonicity.

Thus we conclude the following:

The algorithm (15) or (16) converges to the unique fixed point (12), if it exists, under the conditions that

$$I_i < \frac{c_i g_{ii} \gamma_i^{tar}}{b_i} \tag{29}$$

and

$$p_i \leq \frac{c_i (\gamma_i^{tar})^2}{2b_i} \tag{30}$$

The condition (28) can be written in an equivalent form (since SIR must be nonnegative)

$$\gamma_i^{tar} \geq \sqrt{\frac{2b_i p_i}{c_i}}. \tag{31}$$

These conditions seem very natural since practical applications require an upper limit on interference power and a lower limit on target SIR. For example, assuming $b_i = c_i = 1$, we have the very simple limiting conditions, $I_i < g_{ii}\gamma_i^{tar}$, $p_i < 0.5\gamma_i^{tar}$, and $\gamma_i^{tar} \geq \sqrt{2p_i}$, that must be satisfied to assure convergence to the unique Nash equilibrium.

D. Existence of A Nash Equilibrium

In this section we establish the existence of a solution for the Nash algorithm algebraic equations under the same condition that guarantees the existence of the unique solution for the power balancing equations. The result is established by using the Implicit Function Theorem (see for example, [42], page 128). Using (4) in (10), the considered system of algebraic equations is given by

$$\begin{aligned} 0 &= -p_i + \frac{\gamma_i^{tar}}{g_{ii}} \left(\sum_{j \neq i} g_{ij} p_j + \eta_i \right) - \frac{b_i}{2c_i g_{ii}^2} \left(\sum_{j \neq i} g_{ij} p_j + \eta_i \right)^2 \\ &= F_i(p_i, \mathbf{p_{-i}}, g_{ii}, g_{ij}, g_{ij}^2, b_i, c_i, \eta_i), \\ &\qquad i, j = 1, 2, ..., N, \ i \neq j. \end{aligned}$$

According to the Implicit Function theorem the Jacobian matrix (the matrix of partial derivatives $\frac{\partial F_i}{\partial p_i}$ must be non-singular at the point of existence. Note that in the case of power balancing, the corresponding algebraic equations are represented by the first two terms of the right-hand side of the above formula so that the corresponding Jacobian matrix has -1 on the main diagonal and $\gamma_i^{tar} g_{ij}/\gamma_{ii}$ outside the main diagonal. When this Jacobian matrix is nonsingular then the power balancing solution exists. (It is customary in power control literature to say that the solution is feasible, that is, existence and feasibility have the same meaning).

Note that the solution mostly exists owning to the fact that g_{ij} are very small quantities (order of 10^{-3} and smaller). Note also that the receiver noise, η_i, is also very small in practice (order of 10^{-10}) or so. The Jacobian matrix for the Nash algorithm equations given above will have in addition (to the Jacobian matrix of power balancing) the terms proportional to g_{ij}^2, $g_{ij}\eta_i$, and η_i^2 (all of them coming from the third term in the above system of algebraic equations). These terms are all extremely small and will not have an impact on nonsingularity of the corresponding Jacobian matrix. Even more, the Jacobian matrices in both cases are continuous functions with respect to g_{ij}, which further implies ([42], page 128) that the solutions exist for an entire range of small values g_{ij}. In conclusion, assuming that g_{ij} are small enough (with the standard of smallness determined by the existence of the power balancing solution), then the solution of the defined Nash algebraic equations exists.

E. Opting Out

Note that *there may still be systems for which no practical Nash solution is feasible*. If the system is too heavily loaded, the static Nash algorithm, like the power balancing algorithm, may yield unacceptably low SIR's. Of course, those mobiles whose SIR's fall below a minimum QoS threshold should be dropped because otherwise they cause unnecessary interference to other mobiles using the same frequency channel. Optimal strategies for choosing when to drop calls and which calls to drop are beyond the scope of this paper. This related research topic is not specific to the Nash algorithm but rather arises regardless of the algorithm used. For call-dropping strategies for algorithms including but not limited to the power-balancing algorithm, the interested reader is referred to [6], [14], [25], [43], [44], [45], and [46]. Note that a simple call dropping mechanism: drop mobile i if $\gamma_i^{(k)} < \delta\gamma^{tar}$, $0 < \delta < 1$, can be easily incorporated in our algorithm.

A significant advantage of the proposed algorithm is that an individual mobile may choose to stop transmitting rather than continue to expend power to achieve unsatisfactory QoS. If we return to the two versions of the algorithm,

$$p_i^{(k+1)} = \frac{\gamma_i^{tar}}{g_{ii}} I_i^{(k)} - \frac{b_i}{2c_i} \left(\frac{I_i^{(k)}}{g_{ii}} \right)^2 \tag{32}$$

and

$$p_i^{(k+1)} = \gamma_i^{tar} \left(\frac{p_i^{(k)}}{\gamma_i^{(k)}} \right) - \frac{b_i}{2c_i} \left(\frac{p_i^{(k)}}{\gamma_i^{(k)}} \right)^2 =: f_i(p_i^{(k)}) \tag{33}$$

we can derive the conditions under which the mobile stops transmitting or "opts out". We will find that these conditions are related to the limits determined above in showing convergence of the algorithm.

Although the two versions of the algorithm are equivalent in the sense that one can be obtained from the other using the definition of the SIR, there is a slight difference. The interference-based form can start from zero initial power since the noise term of the interference is always nonzero. The power-based form, however must start from a nonzero initial power. We can use this to our advantage, by starting with the interference-based form of the algorithm, then switching after a few iterations to the power-based form.

Let's examine (32). If the interference to the mobile's transmission is too large, namely

$$I_i^{(k)} > \frac{2c_i g_{ii} \gamma_i^{tar}}{b_i} \tag{34}$$

then the power will be set to zero. (This was the restriction needed to achieve positivity of the update function.) If we use the second form of the algorithm to update the power, we see that once the interference exceeds this threshold and power must be set to zero. In order to reestablish communication, the mobile would have to restart using the interference-based form, presumably only after detecting that the interference level has decreased to below the threshold.

We used the framework of Yates to show that the algorithm converges for a limited range of interference values (29). The question arises whether we can utilize the algorithm outside this range of values or whether we must apply some sort of ad-hoc limits when the value of the interference is outside this range. Let us consider interference values in the range

$$\frac{c_i g_{ii} \gamma_i^{tar}}{b_i} < I_i^{(k)} < \frac{2 c_i g_{ii} \gamma_i^{tar}}{b_i} \quad \forall i. \tag{35}$$

If $I_i^{(k)}$ is greater than the upper bound, power is set to zero so there's no convergence problem in that region. Between the two bounds, the function $f_i(p_i^{(k)})$ is positive but not monotonically increasing. Specifically, it is decreasing. In this region $p_i^{(k)}$ decreases for all mobiles so that the interference decreases for all mobiles. Yates' result [21] involves an application of a more general asynchronous convergence theorem [47]. This theorem can also be applied to the sequence of decreasing powers instead of increasing powers. In the case that the individual interferences are simultaneously within this region, (35), for all mobiles, this more general formulation can be used for decreasing powers until the interference has decreased to below the monotonicity bound. Hence, the proposed algorithm may exhibit desirable convergence behavior even when the bounds established in (29) are not met.

III. Simulation

To illustrate the advantages of the proposed algorithm, we compare the Nash equilibrium results with the power balancing (also called SIR-balancing) results. The power balancing algorithm iteratively updates power according to

$$p_i^{(k+1)} = \left(\frac{\gamma_i^{tar}}{\gamma_i^{(k)}}\right) p_i^{(k)}. \tag{36}$$

Note that since admission control is not the subject of our study, we have not implemented any call-dropping algorithm for dropping mobiles whose target SIR cannot be achieved. For the same reason, we have not investigated the effects of changing code length or target SIR. We also did not study the effects of choice of b_i and c_i, since we are only attempting to demonstrate, as opposed to quantify, the potential of the algorithm.

We considered a 2 km square cell with base station centered at the origin and mobile locations were chosen randomly from a uniform distribution[3]. A typical cell is shown in Figure 2. Power was limited to 600 mW corresponding to the legal limit in the US. Background receiver noise power within the user's bandwidth of $\eta_i = 2 \times 10^{-13}$ mW was used in the simulations. The channel gain was determined according to

$$h_i = \frac{A}{r^\alpha} \tag{37}$$

[3]Realistic values for this simulation example were provided by Dr. Larry Greenstein, formerly of AT&T Bell Laboratory and currently at WINLAB, Rutgers University.

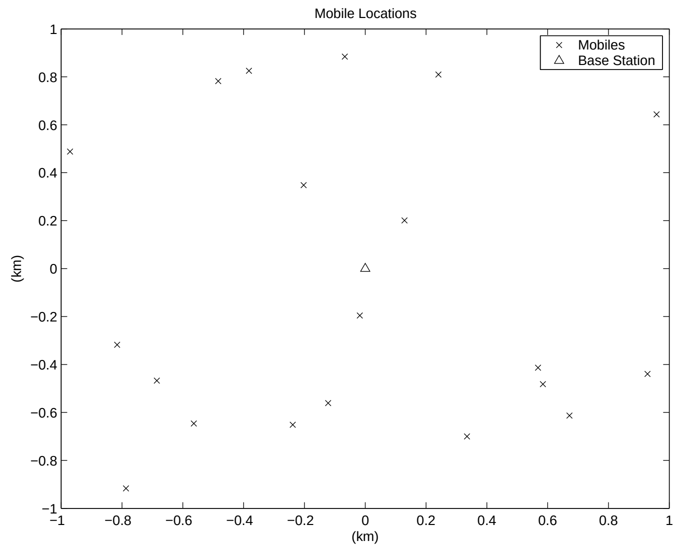

Fig. 2. A typical random distribution of 20 mobiles

where r is the distance from the ith mobile to the base station $\alpha = 4$, and $A = 10^{-11}$, corresponding to a path loss of 110 dB at a distance of 1 km. We used random spreading sequences of length 128. We ignored fast fading and shadow fading, and interference from adjacent cells.

Typical Fully Loaded Cell

In this example, we used twenty mobiles whose locations were chosen uniformly at random within the cell. The locations were shown in Figure 2. Our initial power for all mobiles was $p_i^{(0)} = 0$ for the Nash algorithm (implementing algorithm (30)) and $p_i^{(0)} = 2.22 \times 10^{-16}$ for the power balancing algorithm. We arbitrarily defined convergence as reaching values within 0.01 percent of the steady state values. We found that the Nash algorithm converged in fewer iterations than the power balancing algorithm (29 *vs.* 51), while achieving, on average, a 42 percent reduction in power (final average power $\overline{p}_i^{Nash} = 0.401$ mW versus $\overline{p}_i^{PB} = 0.692$ mW) with just a five percent reduction in average SIR, $\overline{\gamma}_i^{Nash} = 4.78$ as opposed to $\overline{\gamma}_i^{PB} = 5.0 = \gamma^{tar}$. The simulation results are compared in Figure 3 for $b_i = 5$ and $c_i = 1$ for all mobiles.

Impact of Noise

We tested the algorithm in MATLAB simulation with $\gamma_i^{tar} = 5.0$ and $b_i = 0.5$ (mW)$^{-1}$ and $c_i = 1$ for a random configuration of 20 users and noise power levels between 10^{-13} and 10^{-11} mW. As shown in Figure 4, the average powers computed by the algorithm provide significant savings in high noise environments. In these tests, the Nash algorithm always converged in the same number as or fewer iterations than the power balancing algorithm.

Effect of Increasing Number of Users

First, we generated a random set of 35 users and compared the performance of the static Nash and power balancing algorithms for the entire set, as well as subsets consisting of the

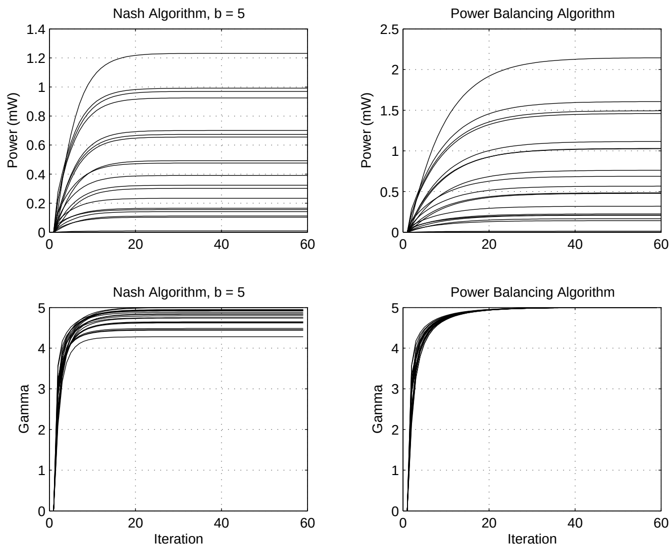

Fig. 3. Performance comparison of Nash and power balancing algorithms for 20 users with $\gamma_i^{tar} = 5.0$ for all mobiles

first 15, 20, 25, and 30 users in the set. The results are shown in Figure 5. The traces represent the average SIR and power values over the set of mobiles considered. It is apparent that with a slight sacrifice of achieved SIR, a significant decrease in power is achieved. However, with a large number of users, the achievable power balancing SIR and the Nash equilibrium SIR are decreased significantly. Accordingly, dropping some mobiles whose SIR fell below the minimum acceptable SIR value would be necessary to achieve QoS targets in practice.

The Opt Out Phenomenon

Examining the performance of 40 individual mobiles in Figure 6 one sees that a few mobiles opt out when their cost of not transmitting becomes less than their cost of transmitting. The value of b was chosen so that the entire range of allowable

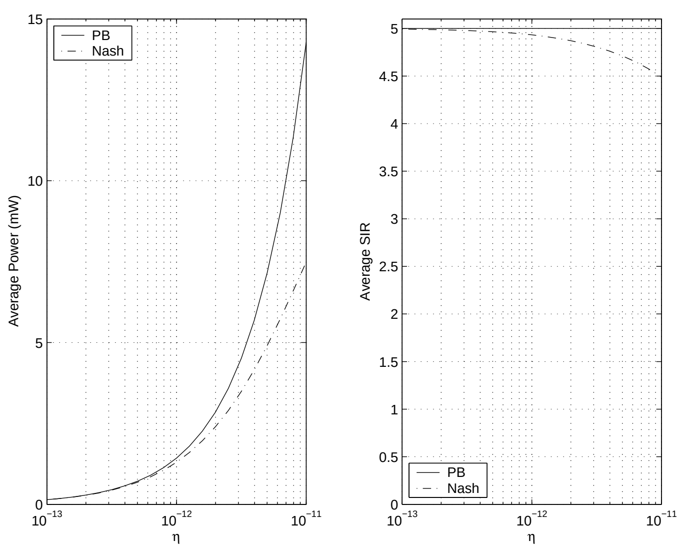

Fig. 4. Performance comparison of Nash and power balancing algorithms for 20 users with target SIR 5, for a range of noise values

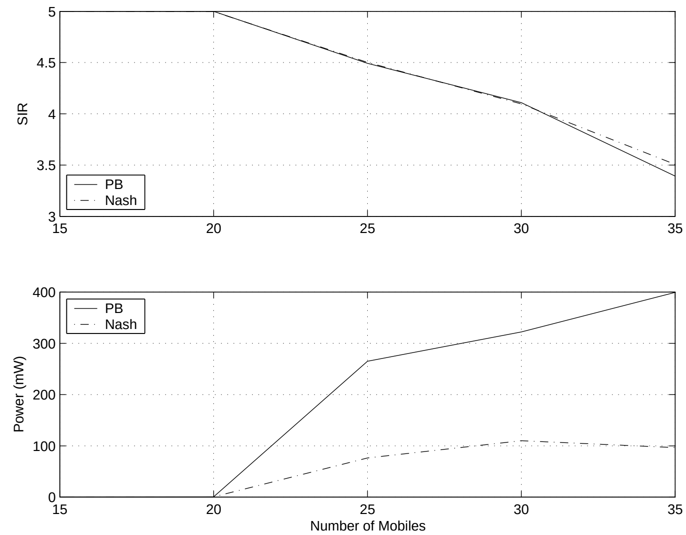

Fig. 5. Average performance comparison of Nash and power balancing algorithms for 15, 20, 25, 30, and 35 users with target SIR 5, $b = 0.05$ (mW)$^{-1}$, $c = 1$, power in units of mW

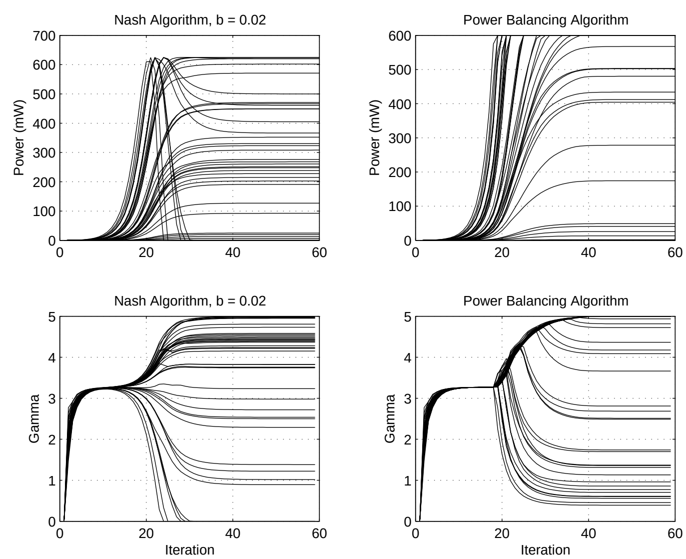

Fig. 6. Performance comparison of Nash and power balancing algorithms for 40 users with target SIR 5, $b = 0.02$ (mW)$^{-1}$, $c = 1$

powers would be used. The average SIR's and powers for this example were both 3.29, using $\overline{p}^{Nash} = 197$ mW and $\overline{p}^{PBA} = 399$ mW. Eliminating the 5 mobiles that opted out, one obtains $\overline{\gamma}^{Nash} = 3.75$ and $\overline{p}^{Nash} = 225$ mW. By comparison, if we eliminate the 5 mobiles having the worst power balancing SIR's and rerun the power balancing algorithm, we obtain $\overline{\gamma}^{PBA} = 3.77$ using $\overline{p}^{PBA} = 359$ mW, and the base station would have to select these mobiles and terminate their calls, whereas using the Nash algorithm, the mobiles themselves chose to opt out based on their own power versus SIR error costs. In this example, the mobiles 10, 12, 16, 23 and 26 opted out, whereas the base station would have dropped mobile 4 instead of mobile 10. Note that if the mobiles used different choices of weighting coefficients b_i and c_i, the set that opted out could even be disjoint from the set chosen by the base station.

In Section II-C we showed that if certain bounds on interference were not exceeded, the algorithm would converge to the nonzero Nash equilibrium power and SIR. What we have seen in the simulations is that, in fact, the algorithm behaves well even when the interference exceeds these bounds.

IV. Conclusion

With our algorithm, we obtained lower individual powers with comparable or faster convergence by compromising slightly on SIR values. Exploiting this tradeoff, the proposed algorithm was able to handle many more users than the power balancing algorithm and to produce the Nash equilibrium in cases where the power balancing problem has no solution. The algorithm can easily be implemented in a distributed manner, and has the advantage that mobiles choose whether or not to transmit based on their own valuations of the trade-offs between power usage and QoS as represented in their cost functions.

An interesting topic for future research is the development of efficient algorithms for use by the base station in identifying when to drop calls and which mobile's calls to drop. Admission and dropping algorithms have received considerable attention in the context of power balancing type algorithms, but has not been investigated for static Nash equilibrium algorithms.

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References

[1] *TIA/EIA Interim Standard: Mobile Station–Base Station Compatibility Standard for Dual–Mode Wideband Spectrum Cellular System*, Telecommunications Industry Association, July 1993.
[2] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communications*. Boston: Addison-Wesley, 1995.
[3] J. M. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Technical Review*, vol. 3, no. 2, pp. 277–299, 1973.
[4] H. J. Meyerhoff, "Method for computing the optimum power balance in multibeam satellites," *COMSAT Technical Review*, vol. 4, no. 1, pp. 139–146, 1974.
[5] R. W. Nettleton and H. Alavi, "Power control for a spread spectrum cellular mobile radio system," in *Proceedings, IEEE Vehicular Technology Conference*, 1983, pp. 242–246.
[6] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Vehicular Technology*, vol. 41, no. 1, pp. 57–62, Feb. 1992.
[7] ——, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Vehicular Technology*, vol. 41, no. 3, pp. 305–311, Aug. 1992.
[8] H. Wang, A. Huang, R. Hu, and W. Gu, "Balanced distributed power control," in *Proceedings, The 11th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 2, 2000, pp. 1415–1419.
[9] B. He, M. Z. Wang, and K. C. Li, "A new distributed power balancing algorithm for CDMA cellular systems," in *Proceedings, 1997 IEEE International Symposium on Circuits and Systems*, 1997, pp. 1768–1771.
[10] C. W. Sung and W. S. Wong, "Performance of a cooperative algorithm for power control in cellular systems with time-varying link gain matrix," *Wireless Networks*, vol. 6, pp. 429–439, 2000.
[11] S. A. Grandhi and J. Zander, "Constrained power control in cellular radio systems," in *Proceedings, IEEE 44th Vehicular Technology Conference*, vol. 2, 1994, pp. 824–828.
[12] N. D. Bambos, S. C. Chen, and G. J. Pottie, "Radio link admission algorithms for wireless networks with power control and active link quality protection," in *Proceedings, INFOCOM '95*, vol. 1, 1995, pp. 97–104.
[13] R. Jäntti and S.-L. Kim, "Selective power control with active link protection for combined rate and power management," in *Proceedings, IEEE 51st Vehicular Technology Conference*, 2000, pp. 1960–1964.
[14] F. Berggren, R. Jäntti, and S.-L. Kim, "A generalized algorithm for constrained power control with capability of temporary removal," *IEEE Trans. Vehicular Technology*, vol. 50, no. 6, pp. 1604–1612, Nov. 2001.
[15] C.-Y. Huang and R. D. Yates, "Rate of convergence for minimum power assignment algorithms in cellular radio systems," *Wireless Networks*, vol. 4, pp. 223–231, 1998.
[16] M. Xiao, N. N. Shroff, and E. K. Chong, "Utility-based power control in cellular wireless systems," in *Proceedings, IEEE INFOCOM 2001*, 2001, pp. 412–421.
[17] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Vehicular Technology*, vol. 42, no. 4, pp. 641–646, Nov. 1993.
[18] R. Jäntti and S.-L. Kim, "Second-order power control with asymptotically fast convergence," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 447–457, Mar. 2000.
[19] Z. Uykan, R. Jäntti, and H. N. Koivo, "A PI-power control algorithm for cellular radio systems," in *Proceedings, IEEE 6th International Symposium on Spread-Spectrum Techniques and Applications*, vol. 2, 2000, pp. 782–785.
[20] F. Berggren, "Distributed power control for throughput balancing in CDMA systems," in *Proceedings, 12th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, 2001, pp. 24–28.
[21] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, Sept. 1995.
[22] K. K. Leung, C. W. Sung, W. S. Wong, and T. M. Lok, "Convergence theorem for a general class of power control algorithms," in *IEEE International Conference on Communications (ICC 2001)*, vol. 3, 2001, pp. 811–815.
[23] S. Grandhi, J. Zander, and R. D. Yates, "Constrained power control," *International Journal of Wireless Personal Communications*, vol. 1, no. 4, 1995.
[24] J. Zhang, E. K. P. Chong, and I. Kontoyiannis, "Unified spatial diversity combining and power allocation for CDMA systems in multiple time-scale fading channels," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 7, pp. 1276–1288, July 2001.
[25] C. Wu and D. P. Bertsekas, "Distributed power control algorithms for wireless networks," *IEEE Trans. Vehicular Technology*, vol. 50, no. 2, pp. 504–514, Mar. 2001.
[26] M. Huang, P. E. Caines, C. D. Charalambous, and R. P. Malhamé, "Power control in wireless systems: a stochastic control formulation," in *Proceedings, American Control Conference*, vol. 2, 2001, pp. 750–755.
[27] ——, "Stochastic power control for wireless systems: Classical and viscosity solutions," in *Proceedings, 40th IEEE Conference on Decision and Control*, 2001, pp. 1037–1042.
[28] H.-J. Su and E. Geraniotis, "Adaptive closed-loop power control with quantized feedback and loop filtering," *IEEE Trans. Wireless Communications*, vol. 1, no. 1, pp. 76–86, Jan. 2002.
[29] T. Alpcan, T. Basar, R. Srikant, and E. Atman, "CDMA uplink power control as a noncooperative game," *Wireless Networks*, vol. 8, pp. 659–670, 2002.
[30] H. Ji and C.-Y. Huang, "Non-cooperative uplink power control in cellular radio systems," *Wireless Networks*, vol. 4, no. 3, pp. 233–240, 1998.
[31] A. B. MacKenzie and S. B. Wicker, "Game theory in communications: Motivation, explanation, and application to power control," in *Proceedings, IEEE GLOBECOM '01*, 2001, pp. 821–826.
[32] V. Shah, N. B. Mandayam, and D. J. Goodman, "Power control for wireless data based on utility and pricing," in *Proceedings, Ninth IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 3, 1998, pp. 1427–1432.

[33] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Pareto efficiency of pricing-based power control in wireless data networks," in *Proceedings, IEEE Wireless Communications and Networking Conference, 1999*, vol. 1, 1999, pp. 231–235.

[34] ——, "Power control in a multicell wireless data system using pricing," in *Proceedings, IEEE 52nd Vehicular Technology Conference*, vol. 2, 2000, pp. 484–491.

[35] ——, "Pricing and power control in a multicell wireless data network," *IEEE Journal on Selected Areas in Communications*, vol. 19, pp. 1883–1892, Oct. 2001.

[36] ——, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Communications*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[37] T. Başar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed. Philadelphia, PA: SIAM, 1999.

[38] A. Ramesh and A. Chockalingam, "Performance analysis of adaptive modulation with optimum power control in cellular systems," in *Proceedings, IEEE International Conference on Personal Wireless Communications*, 2000, pp. 193–197.

[39] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Transactions on Wireless Communications*, vol. 1, no. 1, pp. 46–55, Jan. 2002.

[40] J. Nash, "Noncooperative games," *The Annals of Mathematics, 2nd ser.*, vol. 54, pp. 286–295, 1951.

[41] Z. Gajic and S. Koskie, "Newton iteration acceleration of the Nash game algorithm for power control in 3G wireless CDMA networks," in *Proceedings, ITCOM 2003, Conference on Performance and Control of Next Generation of Communication Networks*, 2003, pp. 115–121.

[42] J. M. Ortega and W. C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*. New York: Academic Press, 1970.

[43] R. D. Yates, S. Gupta, C. Rose, and S. Sohn, "Soft dropping power control," in *Proceedings, IEEE 47th Vehicular Technology Conference*, vol. 3, 1997, pp. 1694–1698.

[44] D. Kim, K.-N. Chang, and S. Kim, "Efficient distributed power control for cellular mobile systems," *IEEE Transactions on Vehicular Technology*, vol. 46, no. 2, pp. 313–319, May 1997.

[45] Q. Wu, "Performance of optimum transmitter power control in CDMA cellular mobile systems," *IEEE Trans. Vehicular Technology*, vol. 48, no. 2, pp. 571–575, Mar. 1999.

[46] M. Andersin, Z. Rosberg, and J. Zander, "Gradual removals in cellular PCS with constrained power control and noise," *Wireless Networks*, vol. 2, pp. 27–43, 1996.

[47] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation*. Englewood Cliffs, NJ: Prentice Hall, 1989.

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