SIR-Based Power Control Algorithms for Wireless CDMA Networks: An Overview *

S. Koskie Z. Gajic

Wireless Information Network Laboratory (WINLAB)
Rutgers University
Piscataway, NJ 08854
{koskie,gajic}@winlab.rutgers.edu

Abstract— This paper summarizes and explains the main results on signal to interference (SIR) based power control algorithms, which are used to increase capacity and improve quality of service in cellular wireless radio systems. The classic works of Aein, Meyerhoff, Nettleton, and Alavi attracted considerable attention in the nineties. The modern approach to the power balancing control problem in wireless networks, formulated by Zander in 1992, matured in the papers of Foschini and Yates and their coworkers in the latter part of the nineties. However the field is still wide open for research as was indicated in the overview paper by Hanly and Tse (1999). The most recent approaches to solving the mobile power distribution problem in wireless networks use Kalman filters, dynamic estimators, and noncooperative Nash game theory.

I. Background and Conventions

Communication networks can be fixed or mobile; however, the same basic power control problem is common to both types. We do not concern ourselves here with motion of mobile stations. Accordingly, we use the terms "mobile" and "user" interchangeably.

Information travels in two directions in a network: uplink (mobile-to-base) and downlink (base-to-mobile). The mathematical formulations of these problems are similar in satellite communications; they differ in cellular wireless systems. In cellular communications networks, an uplink-specific concern is conservation of mobile battery power. Also, downlink codes are synchronous and can be made orthogonal; but uplink codes arrive at the base station asynchronously, resulting in cross correlation, and hence high in-cell interference potential unless power is adequately controlled. For this reason we concentrate on the uplink.

Three types of multiple access techniques are commonly used, namely, time-, frequency- and code-division multiple access: TDMA, FDMA, and CDMA, respectively. TDMA and FDMA protocols assign a specific time, respectively, frequency slot to each user. These protocols are relatively wasteful in that when a user does not transmit in its assigned slot, no other user can make use of the resource. In contrast, in a CDMA system, all users share the same time-frequency space. In CDMA, the individual signals are distinguished by encoding and decoding using distinct code sequences assigned to each user. The code bandwidth is chosen to be much larger than the signal bandwidth, generating a spread spectrum signal. Most current cellular wireless networks use CDMA or TDMA techniques.

Regardless of the access method, a common physical model is appropriate for use in power control. We characterize the system by a gain matrix G, the meaning of whose entries depends on access method and link direction. In TDMA and FDMA, interference arises from transmissions of users assigned the same slot in nearby cells; thus a matrix entry represents effective path gain between a pair of users. In CDMA, the effective path gain g_{ij} depends on distance and code cross correlation of same-cell users.

We address our analysis specifically to CDMA networks as advantages of spread spectrum techniques include

Capacity Combinations of powerful coding techniques and reuse of frequencies in every cell allows CDMA to provide higher capacity than TDMA.

Privacy The code must be known in order to despread the received signal to recover the information.

Interference rejection and anti-jamming Spread spectrum techniques are effective against both narrow band and wide band interference and jamming.

Two significant interference mechanisms are termed "near-far" and "corner" effects. The *corner effect* is observed in the downlink with the mobile approximately equidistant from three base stations (*i.e.* at a corner of a hexagonal cell). The *near-far effect* dominates in the uplink. When mobiles transmit with equal power, the signals of mobiles close to the base station interfere strongly with those of mobiles far from the base station.

Path loss, modeled using an inverse power law relationship, is the main phenomenon that determines the values of the gain matrix entries. Spatially averaged received power, p_{rec}, at a point located a distance r from a transmitter is

$$p_{rec} \propto \frac{p_{trans}}{r^{\alpha}} \tag{1}$$

where p_{trans} is the transmitted power and the path loss exponent α is two in free space, somewhat higher in indoor

*This work was supported by National Science Foundation grant ANIR-0106857.

systems[1], and generally in the range of three to five for outdoor networks [1].

Two additional phenomena that affect transmissions are shadowing and fast fading. The presence of obstructions such as buildings, hills and trees causes slow multiplicative gain fluctuations called shadowing, which can be thought of as changing the user's effective position. Fast multipath fading results from signal reflections whose relative phases change with user motion. Assuming many multipaths and ideal Rake processing, these fluctuations can be ignored. As the effects of unmodeled shadow fading and fast fading do not change the general nature of the power control problem, we use the inverse power law model (1).

II. History and Background

Cochannel interference resulting from frequency reuse is a major factor limiting network capacity in cellular radio systems, so a power control algorithm that reduces cochannel interference has the potential to increase network capacity. Because each user contributes to the interference affecting other users, effective and efficient power control strategies are essential for achieving both quality of service (QoS) and system capacity objectives.

The need for dynamic control of transmitted power in spread spectrum mobile communication systems was first encountered in the area of satellite communications. To fill this need, SIR balancing (also called power balancing) algorithms were proposed by Aein [2] and Meyerhoff [3] in the early 1970's. A decade later, their results were adapted by Nettleton and Alavi [4], [5], and [6] for spread spectrum mobile cellular systems. The power balancing algorithms equalize, where possible, the SIR's of all users. Although communication systems are stochastic, the power control problem leads to a purely deterministic eigenvalue problem or a linear equation, as we shall see below.

Open-loop power control in wireless networks has been employed to combat path loss and shadow fading [7]. The average power control techniques of Gilhousen *et al.* [8] and Viterbi *et al.* [9] maintain received local mean constant, mitigating the effect of shadowing and near-far effects.

Closed loop power control is used in wireless communication networks to compensate for fast fading and time-varying channel characteristics, and to reduce mobile battery power consumption. The closed loop control structure in IS-95 (a currently implemented CDMA standards used in wireless networks) consists of an outer loop algorithm that updates the SIR threshold every 10 ms and an inner loop which, based on SIR measurements, updates required powers at 800 Hz [10]. The algorithms that we present would replace the inner loop control algorithm. They would require additional power control bits if centrally implemented, but have the advantage that they could be implemented in a distributed manner, with the inner loop control algorithm implemented at the mobile rather than the base station. In this case, the mobile would need SIR and target SIR signals transmitted from the base station.

[1]In indoor systems, the path loss exponent depends on the degree of clutter and presence or absence of a line of sight (LoS) path.

III. Power Control as an Eigenvalue/Eigenvector Problem

This section will show that if the effect of noise power on the interfering signal experienced is sufficiently small to be neglected, formulating the power control problem mathematically leads to an eigenvalue problem involving positive matrices. Results on nonnegative matrices can be found in Gantmacher [11], Minc [12], or Varga [13].

If we let γ_d and γ_u denote the desired downlink and uplink SIR's, respectively, for all users, Nettleton and Alavi [5] showed that the corresponding balanced power vectors, $\mathbf{p}_d$ and $\mathbf{p}_u$ must satisfy the eigenvalue problems

$$G\mathbf{p}_u = \frac{1+\gamma_u}{\gamma_u}\mathbf{p}_u \quad \text{and} \quad G^T\mathbf{p}_d = \frac{1+\gamma_d}{\gamma_d}\mathbf{p}_d \tag{2}$$

where the matrix G is a nonnegative matrix of known parameters whose size depends on the number of mobiles of each cell and whose entries depend on the distances from each user to each base station. (Elements of G can be thought of as effective gains.)

From (2) we can see that with $\lambda(G)$ denoting the eigenvalues of G, a solution to the SIR balancing problem, if it exists, is

$$\frac{1+\gamma_u}{\gamma_u} = \frac{1+\gamma_d}{\gamma_d} \in \lambda(G). \tag{3}$$

In the physical power control problem, the powers $\mathbf{p}_d$ and $\mathbf{p}_u$ and the SIR's γ_d and γ_u must all be positive, so (3) shows that a solution exists only if a) the matrix G has a real positive eigenvalue greater than one and b) the corresponding left and right eigenvectors are nonnegative.

We assume without loss of generality that G is irreducible, since if G is not irreducible, it should be decomposed and the subsystems analyzed separately. G then being nonnegative and irreducible, we may apply the Perron-Frobenius theorem to conclude that G has a unique real eigenvalue equal to its spectral radius $\rho(G)$, whose corresponding eigenvector has all components of the same sign. The components can then all be chosen positive.

It then follows from (3) that so long as the spectral radius $\rho(G)$ satisfies $\rho(G) > 1$, one solution to the double link power balancing problem [5] is given by

$$\gamma_u = \gamma_d = \frac{1}{\rho(G)-1}. \tag{4}$$

The corresponding right and left eigenvectors of the matrix G then give the corresponding balanced powers. Of course, the eigenvectors are unique only up to a multiplicative constant, and hence would be chosen, subject to physical constraints, to minimize the power used.

The next task is to show that the spectral radius $\rho(G)$ is indeed greater than one. Consider a cellular wireless system in which n users share a channel. If the effect of noise power is neglected, the interference $I_i(p_{-i})$ to the ith user's signal will be $\sum_{j\neq i} g_{ij}p_j$ where p_i is the transmission power corresponding to the ith user and the g_{ij} are the link gains. We use the subscript "$-i$" to indicate that the power of mobile i does not contribute to the interference it experiences. The SIR for the ith user is thus

$$\gamma_i = \frac{g_{ii}p_i}{I_i(p_{-i})} = \frac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j}, \quad i = 1, 2, \ldots, n. \tag{5}$$

Despite the fact that in a mobile wireless system the link gains change over time, most theoretical studies assume that the link gains are constant.

Definition 1 *If we define the power vector by* $\mathbf{p}^T = [p_1, p_2, \ldots, p_n]$*, the SIR level* γ *is said to be* achievable by a power control algorithm if there exists a power vector $\mathbf{p} > 0$ (*i.e.* $p_i > 0$ for all i) such that $\gamma_i \geq \gamma$ for all i.

For convenience, we normalize the system matrix to be

$$G = \begin{bmatrix} 1 & \frac{g_{12}}{g_{11}} & \frac{g_{13}}{g_{11}} & \cdots & \frac{g_{1n}}{g_{11}} \\ \frac{g_{21}}{g_{22}} & 1 & \frac{g_{23}}{g_{22}} & \cdots & \frac{g_{2n}}{g_{22}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{g_{n1}}{g_{nn}} & \frac{g_{n2}}{g_{nn}} & \cdots & \frac{g_{nn-1}}{g_{nn}} & 1 \end{bmatrix}. \tag{6}$$

A reasonable question at this point is, "In the above described power control system, what SIR values can be achieved?". If the SIR's are required to be equal, the answer was established by both Nettleton and Alavi [5] and Meyerhoff [3] and later restated by Zander [14] in a stochastic framework. The result can be stated as follows:

Lemma 1 *There exists a unique maximum achievable SIR level defined by*

$$\lambda^* = \max\{\gamma \,|\, \exists \mathbf{p} > 0 \text{ such that } \gamma_i \geq \gamma, \forall i\}. \tag{7}$$

The maximum is given by

$$\gamma^* = \frac{1}{\lambda^* - 1}, \quad \lambda^* \neq 1 \tag{8}$$

where λ^* *is the unique real positive eigenvalue of the matrix* G *for which the eigenvector problem*

$$G\mathbf{p} = \lambda^*\mathbf{p} \tag{9}$$

has a solution $\mathbf{p} > 0$*. Such* λ^* *and* $\mathbf{p}$ *exist as* G *is a nonnegative matrix. Also* $\lambda^* = \rho(G)$*, the spectral radius of* G*.*

If the SIR's are not constrained to be equal, the power control problem is not the standard eigenvalue problem, but leads instead to the matrix equation

$$(G - I)\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma_1} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{1}{\gamma_2} & 0 & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \ddots & 0 & \frac{1}{\gamma_{n-1}} & 0 \\ 0 & \cdots & \cdots & 0 & \frac{1}{\gamma_n} \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_n \end{bmatrix}. \tag{10}$$

We will discuss power control methods that do not require all SIR's to be equal in later chapters.

Note that since the SIR is a positive quantity, (8) indicates that the dominant eigenvalue of the matrix G must be not only real and positive, but also greater than one. That this does not require additional restrictions on the matrix G is easily shown using Gerschgorin's theorem, a statement of which can be found in standard linear algebra texts. Recall that the sum of the eigenvalues of a matrix is equal to the sum of the diagonal elements, which is n in the case of the normalized matrix G (6). This implies that $\sum_{i=1}^{n} \lambda_i = n$, hence at least one of them (the unique real one representing the matrix spectral radius) must be greater than one.

A. Iterative Solution: The Power Method

In the 1970's, Meyerhoff [3] proposed finding the balanced power vector using an iterative algorithm that has the form of the *power method*, a classic method of linear algebra. Aein [2] and Meyerhoff [3] also proposed a slight modification for finding the balanced power when background (thermal) noise power is added to the interference power.

Solving the standard eigenvalue problem (9), we obtain the balanced SIR, and the corresponding powers (scaled by a constant factor). Note that solving this eigenvalue problem would require knowledge of information related to all mobiles. Accordingly, the power control would have to be implemented centrally, presumably at the base station. Disadvantages of centralized solutions are that 1) it is difficult to measure the $g_{ij}, i \neq j$ and 2) a solution calculated centrally must then be communicated to the users, resulting in excessive network data flow and computational burden.

On the other hand, the SIR's, which carry information about the g_{ij}'s, can be measured at any time instant. By sampling at discrete time instants k we obtain the SIR estimates $\hat{\gamma}_i^{(k)}$ which satisfy

$$\hat{\gamma}_i^{(k)} = \frac{g_{ii}p_i^{(k)}}{\sum_{j\neq i}^{n} g_{ij}p_j^{(k)}}, \quad i = 1, 2, \ldots, n; \quad k = 0, 1, 2, \ldots \tag{11}$$

If we consider normalized power vectors $\mathbf{p}^{(k)}$, the power method for finding the dominant eigenvalue and corresponding eigenvector iterates

$$\mathbf{p}^{(k+1)} = \frac{1}{\|\mathbf{p}^{(k)}\|_\infty} G\mathbf{p}^{(k)}, k = 0, 1, 2, \ldots \tag{12}$$

where $\|\cdot\|_\infty$ denotes the infinity vector norm, here equal to $\max_i\{p_i\}$. Using (6) and (11) in (12), we find that

$$p_i^{(k+1)} = \frac{1}{\|\mathbf{p}^{(k)}\|_\infty}\left(1 + \frac{1}{\hat{\gamma}_i^{(k)}}\right)p_i^{(k)}, \quad i = 1, 2, \ldots, n; \quad k = 0, 1, 2, \ldots \tag{13}$$

so we see that given the measured $\gamma_i^{(k)}$, $k = 0, 1, 2, \ldots$, and starting with almost any nonnegative initial vector $\mathbf{p}^{(0)}$, the appropriate power can be found.

This is the "quasi-distributed" algorithm of Zander [15]. Here, "quasi" indicates that although only the individual SIR estimate is required, knowledge of the entire power vector is needed in order to find the corresponding infinity norm.

From (13) we see that the sequence of measured or estimated SIR's, if convergent, suffices to determine the balanced SIR γ^B. Assuming that the sequence of SIR measurements converges, we solve (13) for γ^B in terms of $\mathbf{p}^B$,

$$\gamma^B = \frac{1}{\|\mathbf{p}^B\|_\infty - 1} \tag{14}$$

where we have defined $\mathbf{p}^B$ to be the normalized power vector obtained as the limit of the right hand side of (13) as k tends to infinity.

Now that we know that the power method converges under this scenario, the next question is how fast it converges. In fact, the power method is very efficient when the ratio $|\lambda_1|/|\lambda_2|$ is large; otherwise, the power method has slow convergence.

Applying the Gerschgorin theorem to the normalized matrix G in (6) tells us that all the eigenvalues of G are located in discs centered at 1 in the complex plane and that

$$|\lambda_i - 1| \le \sum_{j\neq i}^{n} \left|\frac{g_{ij}}{g_{ii}}\right|, \quad i = 1, 2, \ldots, n. \tag{15}$$

Since in practice, it is generally the case that $g_{ii} \ge g_{ij}$, the radii of these circles are relatively small. Hence, the ratio $|\lambda_1|/|\lambda_2|$ is close to 1 so the power method is not an efficient tool for solving the power control problem of wireless systems.

In this section, we have shown that in the absence of noise, finding the balanced power control is equivalent to finding eigenvalues and eigenvectors for the system matrix. In the next section, we will show how the presence of a noise power contribution in the interference leads to a unique solution for the required power vector.

B. Power Control in the Presence of Noise

In fact, noise is always present in a wireless network, and is not always negligible compared to the interference of the cochannel users. We now reconsider the power control problem, this time with the interference given by $I_i(p_{-i}) := \sum_{j\neq i} g_{ij}p_j + \sigma_i^2$ so that the SIR of the ith mobile is

$$\gamma_i = \frac{g_{ii}p_i}{I_i(p_{-i})} = \frac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j + \sigma_i^2}, \quad i = 1, 2, \ldots, n \tag{16}$$

where p_i is the transmission power for user i, g_{ij} is the link gain, and σ_i^2 is receiver noise (background noise) power.

Again the goal in the power control of wireless systems is that every mobile has SIR above its target value, that is

$$\gamma_i \ge \gamma_i^{tar}, \quad i = 1, 2, \ldots, n. \tag{17}$$

In the deterministic approach to the power control problem defined in (16)-(17), it is assumed that the σ_i are constant, known quantities, usually very small with respect to the interference power. Assuming equalities in (17) and knowledge of all gains g_{ij}, (16) can be represented as a system of linear algebraic equations of the form

$$A\mathbf{p} = \mathbf{b}, \quad \mathbf{p} = [p_1, p_2, \ldots, p_N]^T \tag{18}$$

with the elements of A and $\mathbf{b}$ given by $a_{ii} = 1, a_{ij} = -\gamma_i^{tar} g_{ij}/g_{ii}, j \neq i$, and $b_i = \sigma_i^2 \gamma_i^{tar}/g_{ii}$. This system can be directly solved for the p_i, using, for example, the Gaussian elimination method. However, in the currently implemented IS-95 power control mechanism, mobile i knows only its own SIR, $\gamma_i(k)$, at discrete-time instants. In contrast to the eigenvalue problem in which any multiple of the eigenvector produced a balanced solution, the addition of noise power to the interference leads to a problem which will generally have a unique solution.

The literature on numerical linear algebra contains a number of iterative methods for solving systems of linear algebraic equations that can produce the solution of (18) with the knowledge of $\gamma_i(k)$ and γ_i^{tar} only. A number of recent power control algorithms are equivalent to various numerical methods for solving linear equations. We will discuss some of these in subsequent sections.

C. Iterative Techniques for Mobile Power Updates

The power balancing method for solving the power control eigenvalue problem can be solved iteratively using the following algorithm, proposed by Foschini and Miljanic [16], for solving the linear equation (18)

$$\mathbf{p}(k+1) = (I - A)\mathbf{p}(k) + \mathbf{b} \tag{19}$$

where the diagonal terms a_{ii} are one and the off-diagonal terms a_{ij} are given by $a_{ij} = -\gamma_i^{tar} g_{ij}/g_{ii}$, and the elements of $\mathbf{b}$ are $b_i = \sigma_i^2 \gamma_i^{tar}/g_{ii}$. Noting that the diagonal elements of the gain matrix A are all one, we can see that (19) constitutes Jacobi iterations[2] a technique used for iterative solution of linear systems of algebraic equations.

[2]Foschini and Miljanic [16] did not derive (19) as Jacobi iterations. Instead, they assigned power evolution dynamics to cause the steady state value of the dynamic system (described either by differential or difference equation) to be the solution of the linear equation $A\mathbf{p} = \mathbf{b}$. They achieve this by using the following differential or difference equations, respectively, to model the mobile power dynamics

$$\frac{d\mathbf{p}(t)}{dt} = -\alpha\left(A\mathbf{p} - \mathbf{b}\right), \tag{20}$$

$$\mathbf{p}(k+1) = \left(I - \beta A\right)\mathbf{p}(k) + \beta\mathbf{b}, \tag{21}$$

with α being a large positive number and β chosen sufficiently small that $\beta \max_i |\lambda_i(A)| < 1$. It is assumed that the matrix A is antistable, *i.e.* has all eigenvalues in the right half plane, which is the case when A is diagonally dominant. Under this condition, the steady state solutions of (20), respectively, (21) are found to satisfy

$$0 = -\alpha\left(a\mathbf{p}_{ss} - \mathbf{b}\right) \quad \Rightarrow \quad A\mathbf{p}_{ss} = \mathbf{b}$$
$$\mathbf{p}_{ss} = \left(I - \beta A\right)\mathbf{p}_{ss} + \beta\mathbf{b} \quad \Rightarrow \quad A\mathbf{p}_{ss} = \mathbf{b} \tag{22}$$

which satisfy (18) and hence represent the required solution to the power control problem. Foschini and Miljanic derived (20) through the use of what they called a "surrogate" derivative. Note that for $\beta = 1$, (21) is equivalent to (18), and leads to (23), the distributed power control (DPC) algorithm of Grandhi *et al.* [17].

By eliminating the interference, the Jacobi iterations (19) can be reformulated as

$$p_i(k+1) = \frac{\gamma_i^{tar}(k)}{\gamma_i(k)} p_i(k), \quad i = 1, 2, \ldots, n. \tag{23}$$

This algorithm can be implemented distributively since at each time step the ith user needs only a measurement of its own SIR $\gamma_i(k)$ and power $p_i(k)$ to compute its new power command. This algorithm is called the distributed power control (DPC) algorithm [17].

Looking back at the matrix equation (19), we see that the algorithm converges only if the spectral radius (largest eigenvalue magnitude) of the matrix $I - A$ is less than one. If we use Jacobi iterations, the requirement is then that for a suitable choice of the parameter β, the spectral radius of the matrix $I - \beta A$ be less than one.

Since in a real communication system, physical limits constrain the transmit powers to be less than some maximum which we will call p_i^{max}, i.e.

$$0 \leq p_i \leq p_i^{max}, \quad i = 1, 2, \ldots, n, \tag{24}$$

algorithm (23) is then modified accordingly [18], [19] to obtain

$$p_i(k+1) = \min\left\{ p_i^{max}, \frac{\gamma_i^{tar}}{\gamma_i(k)} p_i(k) \right\}, \quad i = 1, 2, \ldots, n. \tag{25}$$

This algorithm is called the distributed constrained power control (DCPC) algorithm and its convergence properties have been studied by Yates [19].

If all gains in (6) are known, the problem of determining appropriate powers to achieve $\gamma_i \geq \gamma$ for all i can be solved by linear programming methods, as pointed out by Bock and Epstein [20] in a different context. More recently, Wu and Bertsekas [21] have presented an integer programming solution and Ramesh and Chockalingam [22] have presented a dynamic programming solutions to the power control problem.

D. Acceleration Techniques

Convergence speed can usually be improved by a factor of two using Gauss-Seidel iterations instead of Jacobi iterations, but this may not suffice in a real-time application. Fixed-point algorithms such as (23) are usually slow to convergence to the solution of the linear equation (18) because, in general, fixed-point algorithms have a linear rate of convergence.

One approach to speeding up the convergence is to use successive overrelaxation techniques. Jäntti and Kim [23] applied this method to obtain the following second-order iterative algorithms which they called unconstrained second-order power control (USOPC):

$$\begin{aligned} p_i(k+1) &= \omega(k) \frac{\gamma_i^{tar}}{\gamma_i(k)} p_i(k) + (1 - \omega(k))\, p_i(k-1), \\ &\quad i = 1, 2, \ldots, n \end{aligned} \tag{26}$$

and constrained second-order power control (CSOPC)

$$\begin{aligned} p_i(k+1) = \min\left\{ p_i^{max},\ \max\left\{0,\ \tilde{p}(k)\right\}\right\}, \\ i = 1, 2, \ldots, n \end{aligned} \tag{27}$$

where

$$\tilde{p}(k) := \omega(k) \frac{\gamma_i^{tar}}{\gamma_i(k)} p_i(k) + (1 - \omega(k))\, p_i(k-1). \tag{28}$$

In (26) and (27) the relaxation parameter $\omega(k)$ has to be appropriately determined at each step of the iteration.

Another approach, recently presented by Li and Gajic [24] uses Steffensen iterations to accelerate the DCPC algorithm (23). The Steffensen method can improve the convergence of an underlying fixed-point algorithm by an order of magnitude. A different acceleration scheme is proposed by Leung [25]. A third scheme which uses projection onto convex sets is proposed by Rabee *et al.* [26]. It has been shown recently in [27] how to accelerate iterative power control update algorithms using Gauss-Seidel iterations.

IV. Additional Literature Highlights

Having discussed the general nature of the power control problem, we now summarize the various additional results found in the literature.

A. Adaptive Step Size for Power Increments

In current systems, uplink power control is achieved by sending a single bit (whose values represent the commands to increase or decrease power by one unit, 1 dB of power) to the mobile. Accordingly, some of the power control literature deals with choice of step-size and or potential improvement achievable if more than one bit is used to send the power control command.

Ariyavisitakul and Chang [28] studied the performance of feedback power control in CDMA systems over fading channels. They concluded that fixed-step power control can perform "nearly as well as a variable-step method and is robust against return channel errors."

Herdtner and Chong [29] discussed convergence of algorithms in which step size is proportional to current power. Kim proposed a quantized power control algorithm that could be implemented using a single power control bit by scaling the power by δ if the PCB is one and δ^{-1} otherwise and studied its convergence. Sung and Wong [30] proposed a distributed fixed-step power control algorithm that, incorporates an additional option to leave the power unchanged if the power is within an asymmetric region defined using the same scale factor as the quantization. In [31] they propose an adaptive threshold for fixed-step power control.

Sim, *et al.* [32] analyzed IS-95 power control, concluding that a 2 dB step is optimal for fixed-step-size algorithms. They also compared fixed and variable step size algorithms.

B. Effects of Processing and Propagation Times

Chockalingam *et al.* [33] studied the effects of measurement updates and sample delay on the performance of a simple closed loop controller.

C. Shaping Dynamics

In addition to those mentioned above, algorithms that shape the dynamics of the controlled power or the algorithm convergence can be found in [34], [16], [23], and [35].

D. Centralized and Decentralized Control Schemes

A centralized controller calculates appropriate control actions for all users based on information such as the link gains. In contrast, a distributed controller must use only locally available information to find a rational control action for a local user. Zander [14], [15] and Li and Gajic [36] proposed centralized SIR-based power control schemes, whereas Grandhi *et al.* [37], Zander [15], Grandhi *et al.* [17] [18], Foschini and Miljanic [16], and Yates [19] proposed distributed controllers. The distributed algorithms of Zander, Grandhi *et al.*, and Foschini and Miljanic need only the user's own SIR to calculate the user's required power.

E. Algorithm Convergence

Foschini and Miljanic showed that their distributed power control algorithm converged synchronously, and Mitra [38] proved its asynchronous convergence. A generalized framework for demonstrating convergence of the uplink power control was proposed by Yates [19], see also Huang and Yates [39] and extended by Leung *et al.* [40]. A stochastic version of the distributed power control algorithm is considered under the assumption that the receiver (background) noise is Gaussian in Ulukus and Yates [41].

F. Call Admission and Call Dropping

Bambos and Pottie [42] explain the power control problem and its relation to the call admission problem. They also point out the need for solving the time-varying power control problem: as the number of active mobiles changes in time, even the order of the problem is time varying. They note that random fluctuations due to shadowing and ducting adversely affect the worst case interference condition for TDMA systems. These have relatively little impact on the capacity of CDMA systems with power control. In the recent paper [43], an admission power control algorithm is developed that maintains active link quality while maximizing free capacity for new admissions.

The *outage probability* is the probability that some randomly chosen mobile has SIR below a prespecified value. The power distribution algorithm proposed in Zander [15] is "optimal" in the sense of minimizing the outage probability. More precisely, it reduces the outage probability to zero by keeping all SIR's above the given threshold, called the *system protection ratio*). Zander, [15] observes that "one somewhat surprising way to minimize the [outage] probability [is] to construct smaller and smaller balanced systems by removing cells (i.e., by turning their transmitters off)." This result should not be surprising as it is a direct consequence of the Perron-Frobenius theorem. Reducing any entry in the matrix G to zero decreases the spectral radius of the matrix, and by (8), increases the corresponding SIR.

Wu [44] shows that, whenever the minimum required SIR is not achieved, mobiles' outage probability can be dramatically decreased, and suggested dropping that mobile whose removal most improves the remaining SIR's. A distributed power control algorithm with active link protection was recently studied by Bambos *et al.* [45].

G. Combatting Fading

In the IS-95B and CDMA2000 standards, fast closed-loop power control is used to fight medium to fast fading. Ariyavisitakul and Chang [28] and Ariyavisitakul [46] show that a higher power control rate can partially accommodate the effect of fast fading. Several closed-loop power algorithms proposed in Chockalingam *et al.* [33] have the ability to compensate for the time-varying channel characteristics. Herdtner and Chong [29] proposed a simple asynchronous distributed power control scheme based on IS-95 [10] and gave the corresponding convergence condition. Song *et al.* [47], [48] and Gunnarson *et al.* [49], [50] have considered up/down power control and closed-loop power control with other nonlinear elements within the framework of nonlinear control systems. In addition, Song *et al.* [47] gave guidelines for choosing the appropriate power control step size in IS-95. Gunnarson *et al.* [49] designed PID controllers to overcome the effects of time-delay in the feedback loop.

H. Stochastic Power Control Formulation

Ulukus and Yates [41] studied the stochastic power control problem formulation using matched filters and assuming white Gaussian noise. Mitra and Morrison [51] proposed a distributed algorithm that adapts mobile power on the basis of local measurements of mean and variance of the interference. They introduced a probabilistic QoS specification, developed an asymptotic framework for estimating orders of magnitude of quantities involved, and gave a condition for geometric rate of convergence.

Leung [52] proposed a power control scheme for TDMA data service based on the Kalman filtering technique. The Kalman filter is used for integrated power control and adaptive modulation/coding in wireless packet-switched networks in Leung and Wang [53]. In Choe *et al.* [54], a linear prediction of received power is made at the base station to predict the power control bit one step ahead. Both approaches assume that the interference is Gaussian.

Qian [55] and Qian and Gajic [56], use an estimator-based optimization approach to solve the stochastic mobile power update problem for wireless CDMA systems, imposing no assumption on the stochastic nature of the interference.

I. Optimization

Some algorithms seek to solve a static optimization problem. The well known distributed constrained power control (DCPC) algorithm maximizes the minimum attained user SIR subject to maximum power constraints [57]. Other algorithms minimize power consumption in the presence of large-scale fading [58] or over a set of discrete available power levels [21].

Dynamic optimization has been used to minimize power consumption by formulating power control for log-normal fading channels in a stochastic framework [59] and [60] as well as to adaptively optimize quantization of SIR error measurements [61].

J. Nash Games Approach

It has been shown in [62] that the power control problem in wireless CDMA can be solved as a Nash game problem with equilibrium solutions requiring less power than the power balancing solution in the case where a tradeoff between SIR error and power usage is permitted. In [63], the unique Nash equilibrium is obtained for the cost function that is the difference between pricing and utility functions.

V. Conclusion

One of the most common approaches to closed-loop power control in wireless communication networks is SIR balancing, also called power balancing. The SIR balancing solution was originally derived for satellite communications by Aein [2] and Meyerhoff [3], and adapted for wireless communications by Nettleton [5] and Zander [14] and [15]. Variations on the SIR balancing algorithm have replaced the target SIR by functions incorporating minimum allowable SIR [64], SIR's of other mobiles [65] and [31], and maximum allowable power [57] and [64] among others. Variations have been developed to incorporate call admission and handoff [66], [67], and [68], base station assignment [39], and economic tradeoffs [69].

SIR balancing algorithms are simple and most can be implemented distributively, but have the disadvantage that convergence can be slow and is guaranteed only if every mobile's target SIR is feasible.

However, we show in [62] that there is still considerable room for improvement in power control algorithms for wireless systems. In that document, we presented a static Nash (noncooperative) game formulation of the power control problem with simulation results indicating that substantial power savings may be achieved in exchange for small deviations in SIR error.

References

[1] R. Prasad, *CDMA for Wireless Pers. Communications.* Boston: Artech House, 1996.

[2] J. M. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Tech. Review*, vol. 3, no. 2, pp. 277–299, 1973.

[3] H. J. Meyerhoff, "Method for computing the optimum power balance in multibeam satellites," *COMSAT Tech. Review*, vol. 4, no. 1, pp. 139–146, 1974.

[4] R. W. Nettleton, "Traffic theory and interference management for a spread spectrum cellular mobile radio system," in *Proc., IEEE ICC-80, Seattle WA*, pp. 24.5.1–24.5.5, 1980.

[5] R. W. Nettleton and H. Alavi, "Power control for a spread spectrum cellular mobile radio system," in *Proc., IEEE VTC.*, pp. 242–246, 1983.

[6] H. Alavi and R. W. Nettleton, "Downstream power control for a spread-spectrum cellular mobile radio system," in *Proc., IEEE Global Telecomm. Conf., Miami FL*, pp. 84–88, Nov. 1982.

[7] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Comm.* Boston: Addison-Wesley, 1995.

[8] K. Gilhousen, I. Jacobs, R. Padovani, A. J. Viterbi, L. Weaver, and C. Wheatley, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Tech.*, vol. 40, pp. 303–312, 1991.

[9] A. J. Viterbi, A. M. Viterbi, and E. Zehavi, "Performance of power-controlled wideband terrestrial digital communication," *IEEE Trans. Comm.*, vol. 41, pp. 559–569, 1993.

[10] Telecomm. Industry Association, *TIA/EIA Interim Standard: Mobile Sation–Base Station Compatibility Standard for Dual–Mode Wideband Spectrum Cellular System*, July 1993.

[11] F. R. Gantmacher, *The Theory of Matrices*, vol. 1. New York: Chelsea Publishing, 1960.

[12] H. Minc, *Nonnegative Matrices.* New York: Wiley, 1988.

[13] R. Varga, *Matrix Iterative Methods.* Berlin: Springer, 2000.

[14] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Tech.*, vol. 41, no. 1, pp. 57–62, 1992.

[15] J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Tech.*, vol. 41, no. 3, pp. 305–311, 1992.

[16] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Tech.*, vol. 42, no. 4, pp. 641–646, 1993.

[17] S. A. Grandhi, R. Vijayan, and D. J. Goodman, "Distributed power control in cellular radio systems," *IEEE Trans. Comm.*, vol. 42, no. 2/3/4, pp. 226–228, 1994.

[18] S. Grandhi, J. Zander, and R. D. Yates, "Constrained power control," *Int. J. of Wireless Pers. Comm.*, vol. 1, no. 4, 1995.

[19] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas in Comm.*, vol. 13, no. 7, pp. 1341–1347, 1995.

[20] F. Bock and B. Epstein, "Assignment of transmitter powers by linear programming," *IEEE Trans. Electromagnetic Compatibility*, vol. 6, pp. 36–44, 1964.

[21] C. Wu and D. P. Bertsekas, "Distributed power control algorithms for wireless networks," *IEEE Trans. Veh. Tech.*, vol. 50, no. 2, pp. 504–514, 2001.

[22] A. Ramesh and A. Chockalingam, "Performance analysis of adaptive modulation with optimum power control in cellular systems," in *Proc., IEEE Int. Conf. Pers. Wireless Comm.*, pp. 193–197, 2000.

[23] R. Jäntti and S.-L. Kim, "Second-order power control with asymptotically fast convergence," *IEEE J. Sel. Areas in Comm.*, vol. 18, no. 3, pp. 447–457, 2000.

[24] X. Li and Z. Gajic, "An improved SIR-based power control for CDMA systems using Steffensen iterations," in *Proc., 2002 Conf. Info. Sci. and Systems*, (Princeton NJ), pp. 287–290, 2002.

[25] Y.-W. Leung, "Power control in cellular networks subject to measurement error," *IEEE Trans. Comm.*, vol. 44, no. 7, pp. 772–775, 1996.

[26] S. A. Rabee, B. S. Sharif, and S. Sali, "Distributed power control algorithm in cellular radio systems using projection onto convex sets," in *Proc., IEEE 54th VTC.*, vol. 2, pp. 757–761, 2001.

[27] D. Lelic and Z. Gajic, "Gauss-Seidel iterations for SIR-based power updates for 3G wireless CDMA communication networks," in *Proc., Allerton Conf. Comm., Control, and Computing*, (Allerton IL), pp. 1348–1357, Oct. 2002.

[28] S. Ariyavisitakul and L. F. Chang, "Signal and interference statistics of a CDMA system with feedback power control," *IEEE Trans. Comm.*, vol. 41, no. 11, pp. 1626–1634, 1993.

[29] J. D. Herdtner and E. K. P. Chong, "Analysis of a class of distributed asynchronous power control algorithms for cellular wireless systems," *IEEE J. Sel. Areas in Comm.*, vol. 18, no. 3, pp. 436–446, 2000.

[30] C. W. Sung and W. S. Wong, "A distributed fixed-step power control algorithm with quantization and active link protection," *IEEE Trans. Veh. Tech.*, vol. 48, no. 2, pp. 553–562, 1999.

[31] C. W. Sung and W. S. Wong, "Performance of a cooperative algorithm for power control in cellular systems with time-varying link gain matrix," *Wireless Networks*, vol. 6, pp. 429–439, 2000.

[32] M. L. Sim, E. Gunawan, C. B. Soh, and B. H. Soong, "Characteristics of closed loop power control algorithms for a cellular DS/CDMA system," *IEEE Proc.*, vol. 145, no. 5, pp. 355–362, 1998.

[33] A. Chockalingam, P. Dietrich, L. B. Milstein, and R. R. Rao, "Performance of closed-loop power control in DS-CDMA cellular systems," *IEEE Trans. Veh. Tech.*, vol. 47, no. 3, pp. 774–789, 1998.

[34] F. Berggren, "Distributed power control for throughput balancing in CDMA systems," in *Proc., 12th IEEE Int. Symp. Pers., Indoor and Mobile Radio Comm.*, pp. 24–28, 2001.

[35] Z. Uykan, R. Jäntti, and H. N. Koivo, "A PI-power control algorithm for cellular radio systems," in *Proc., IEEE 6th Int. Symp. Spread-Spectrum Tech. and Appl.*, vol. 2, pp. 782–785, 2000.

[36] X. Li and Z. Gajic, "Centralized power control in coordinated CDMA systems using Krylov subspace iterations," in *Proc., IASTED Int. Conf. Comm., Internet, and Info. Tech.*, (St. Thomas, Virgin Islands), pp. 305–309, 2002.

[37] S. A. Grandhi, R. Vijayan, D. J. Goodman, and J. Zander, "Centralized power control in cellular radio systems," *IEEE Trans. Veh. Tech.*, vol. 42, no. 4, pp. 466–468, 1993.

[38] D. Mitra, "An asynchronous distributed algorithm for power control in cellular radio systems," in *Wireless and Mobile Communications* (J. M. Holtzman and D. J. Goodman, eds.), pp. 177–186, New York: Kluwer, 1994.

[39] C.-Y. Huang and R. D. Yates, "Rate of convergence for minimum power assignment algorithms in cellular radio systems," *Wireless Networks*, vol. 4, pp. 223–231, 1998.

[40] K. K. Leung, C. W. Sung, W. S. Wong, and T. M. Lok, "Convergence theorem for a general class of power control algorithms," in *IEEE Int. Conf. Comm.*, vol. 3, pp. 811–815, 2001.

[41] S. Ulukus and R. D. Yates, "Stochastic power control for cellular radio systems," *IEEE Trans. Comm.*, vol. 46, no. 6, pp. 784–798, 1998.

[42] N. Bambos and G. J. Pottie, "Power control based on admission policies in cellular radio networks," in *Proc., Global Telecomm. Conf.*, vol. 2, pp. 863–867, 1992.

[43] D. Ayyagari and A. Ephremides, "Power control for link quality protection in cellular DS-CDMA networks with integrated (packet and circuit) services," *Wireless Networks*, vol. 8, no. 6, 2002.

[44] Q. Wu, "Performance of optimum transmitter power control in CDMA cellular mobile systems," *IEEE Trans. Veh. Tech.*, vol. 48, no. 2, pp. 571–575, 1999.

[45] N. Bambos, S. Chen, and G. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE/ACM Trans. Networking*, vol. 8, pp. 583–597, 2000.

[46] S. Ariyavisitakul, "Signal and interference statistics of a CDMA system with feedback power control: Part II," *IEEE Trans. Comm.*, vol. 42, pp. 597–605, 1994.

[47] L. Song, N. Mandayam, and Z. Gajic, "Analysis of an up/down power control algorithm for CDMA reverse link: A nonlinear control approach," in *Proc., Conf. Info. Sys. and Sci.*, (Baltimore MD), pp. 119–124, 1999.

[48] L. Song, N. B. Mandayam, and Z. Gajic, "Analysis of an up/down power control algorithm for the CDMA reverse link under fading," *IEEE J. Sel. Areas of Comm.*, vol. 19, no. 2, pp. 277–286, 2001.

[49] F. Gunnarsson, F. Gustafsson, and J. Blom, "Pole placement design of power control algorithms," in *Proc., IEEE VTC.*, pp. 2149–2153, May 1999.

[50] F. Gunnarsson, F. Gustafsson, and J. Blom, "Improved performance using nonlinear components in power control algorithms," in *Proc., IEEE VTC.*, pp. 1276–1280, May 1999.

[51] D. Mitra and J. A. Morrison, "A distributed power control algorithm for bursty transmissions in cellular spread spectrum wireless networks," in *Wireless Info. Networks: Architecture, Resource Man., and Mobile Data (Proc., 5th WINLAB Workshop, 1995)* (J. M. Holtzman, ed.), pp. 201–212, Kluwer, 1996.

[52] K. K. Leung, "Power control by interference prediction for broadband wireless packet networks," *IEEE Trans. Wireless Comm.*, vol. 1, no. 2, pp. 256–265, 2002.

[53] K. Leung and L. Wang, "Controlling QoS by integrated power control and link adaptation in broadband wireless networks," *European Trans. Telecomm.*, vol. 11, pp. 383–394, 2000.

[54] S. Choe, T. Chulajata, H. M. Kwon, B.-J. Koh, and S.-C. Hong, "Linear prediction at base station for closed loop power control," in *Proc., IEEE 49th VTC.*, vol. 2, pp. 1469–1473, 1999.

[55] L. Qian, *Optimal Power Control in Cellular Wireless Systems.* PhD thesis, Rutgers University, May 2001.

[56] L. Qian and Z. Gajic, "Optimal distributed power control in cellular wireless systems," *Dynamics of Continuous, Discrete and Impulsive Systems*, vol. 10, pp. 537–559, 2003.

[57] S. A. Grandhi and J. Zander, "Constrained power control in cellular radio systems," in *Proc., IEEE 44th VTC.*, vol. 2, pp. 824–828, 1994.

[58] J. Zhang, E. K. P. Chong, and I. Kontoyiannis, "Unified spatial diversity combining and power allocation for CDMA systems in multiple time-scale fading channels," *IEEE J. Sel. Areas in Comm.*, vol. 19, no. 7, pp. 1276–1288, 2001.

[59] M. Huang, P. E. Caines, C. D. Charalambous, and R. P. Malhamé, "Power control in wireless systems: a stochastic control formulation," in *Proc., ACC*, vol. 2, pp. 750–755, 2001.

[60] M. Huang, P. E. Caines, C. D. Charalambous, and R. P. Malhamé, "Stochastic power control for wireless systems: Classical and viscosity solutions," in *Proc., 40th IEEE Conf. on Decision and Control*, pp. 1037–1042, 2001.

[61] H.-J. Su and E. Geraniotis, "Adaptive closed-loop power control with quantized feedback and loop filtering," *IEEE Trans. Wireless Comm.*, vol. 1, no. 1, pp. 76–86, 2002.

[62] S. Koskie, *Contributions to Dynamic Nash Games and Applications to Power Control for Wireless Networks.* PhD thesis, Rutgers University, Jan. 2003.

[63] T. Alpcan, T. Basar, R. Srikant, and E. Atman, "CDMA uplink power control as a noncooperative game," *Wireless Networks*, vol. 8, pp. 659–670, 2002.

[64] H. Wang, A. Huang, R. Hu, and W. Gu, "Balanced distributed power control," in *Proc., The 11th IEEE Int. Symp. Pers., Indoor and Mobile Radio Comm.*, vol. 2, pp. 1415–1419, 2000.

[65] B. He, M. Z. Wang, and K. C. Li, "A new distributed power balancing algorithm for CDMA cellular systems," in *Proc., 1997 IEEE Int. Symp. Circuits and Systems*, pp. 1768–1771, 1997.

[66] N. D. Bambos, S. C. Chen, and G. J. Pottie, "Radio link admission algorithms for wireless networks with power control and active link quality protection," in *Proc., INFOCOM '95*, vol. 1, pp. 97–104, 1995.

[67] R. Jäntti and S.-L. Kim, "Selective power control with active link protection for combined rate and power management," in *Proc., IEEE 51st VTC.*, pp. 1960–1964, 2000.

[68] F. Berggren, R. Jäntti, and S.-L. Kim, "A generalized algorithm for constrained power control with capability of temporary removal," *IEEE Trans. Veh. Tech.*, vol. 50, no. 6, pp. 1604–1612, 2001.

[69] M. Xiao, N. N. Shroff, and E. K. Chong, "Utility-based power control in cellular wireless systems," in *Proc., IEEE INFOCOM 2001*, pp. 412–421, 2001.