Inverting a Matrix using Elementary Row Operations

Our textbook defines three types of elementary row operations:

1. Interchanging rows i and j $j \neq i$ of the matrix: The corresponding elementary row matrix can be obtained from the identity matrix by setting e_{ii} and e_{jj} to zero and setting e_{ij} and e_{ji} to one.
2. Multiplying row i of the matrix by a real number α: The corresponding elementary row matrix can be obtained from the identity by multiplying e_{ii} by α.
3. Adding β times row j to row i $j \neq i$: The corresponding elementary row matrix is obtained from the identity by changing e_{ij} to β.

Example

$$A = \begin{bmatrix} 2 & 5 & 10 \\ 1 & 1 & 1 \\ -2 & -10 & -3 \end{bmatrix}$$

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>> %  Example:  Inverting a matrix using elementary row transformations

>> A = [2 5 10;1 1 1;-2 -10 -3]

A =

     2     5    10
     1     1     1
    -2   -10    -3

>> E1 = [1 0 0; 0 1 0;0 2 1]  % add twice r2 to r3

E1 =

     1     0     0
     0     1     0
     0     2     1

>> aA1 = E1*aA

aA1 =

     2     5    10     1     0     0
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     1     1     1     0     1     0
     0    -8    -1     0     2     1


>> E2 = [1 0 0;-1/2 1 0;0 0 1] % add 1/2 r1 to r2

E2 =

    1.0000         0         0
   -0.5000    1.0000         0
         0         0    1.0000

>> aA2 = E2*aA1

aA2 =

    2.0000    5.0000   10.0000    1.0000         0         0
         0   -1.5000   -4.0000   -0.5000    1.0000         0
         0   -8.0000   -1.0000         0    2.0000    1.0000

>> E3 = [1 0 0;0 1 0;0 -16/3 1] % add -16/3 r2 to r3

E3 =

    1.0000         0         0
         0    1.0000         0
         0   -5.3333    1.0000

>> aA3 = E3*aA2

aA3 =

    2.0000    5.0000   10.0000    1.0000         0         0
         0   -1.5000   -4.0000   -0.5000    1.0000         0
         0         0   20.3333    2.6667   -3.3333    1.0000


>> E4 = [1/2 0 0;0 -2/3 0;0 0 1/aA3(3,3)] % scale all three rows

E4 =

    0.5000         0         0
         0   -0.6667         0
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         0         0   -0.1500

>> aA4 = E4*aA3

aA4 =

  Columns 1 through 5

    1.0000    2.5000    5.0000    0.5000         0         0
         0    1.0000    2.6667    0.3333   -0.6667         0
         0         0    1.0000    0.1311   -0.1639    0.0492

>> E5 = [1 0 -5;0 1 0;0 0 1] % add -5 r3 to r1

E5 =

     1     0    -5
     0     1     0
     0     0     1

>> aA5 = E5*aA4

aA5 =

  Columns 1 through 5

    1.0000    2.5000         0   -0.1557    0.8197   -0.2459
         0    1.0000    2.6667    0.3333   -0.6667         0
         0         0    1.0000    0.1311   -0.1639    0.0492

>> E6 = [1 0 0;0 1 -aA5(2,3);0 0 1]

E6 =

    1.0000         0         0
         0    1.0000   -2.6667
         0         0    1.0000

>> aA6 = E6*aA5

aA6 =

  Columns 1 through 5
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    1.0000    2.5000         0   -0.1557    0.8197   -0.2459
         0    1.0000         0   -0.0164   -0.2295   -0.1311
         0         0    1.0000    0.1311   -0.1639    0.0492

E7 =

    1.0000   -2.5000         0
         0    1.0000         0
         0         0    1.0000

>> aA7 = E7*aA6

aA7 =

  Columns 1 through 5

    1.0000         0         0   -0.1148    1.3934    0.0820
         0    1.0000         0   -0.0164   -0.2295   -0.1311
         0         0    1.0000    0.1311   -0.1639    0.0492

>> E = E7*E6*E5*E4*E3*E2*E1

E =
   -0.1148    1.3934    0.0820
   -0.0164   -0.2295   -0.1311
    0.1311   -0.1639    0.0492


>> iA = aA7(1:3,4:6) % finally, the inverse of A is ...

iA =
   -0.1148    1.3934    0.0820
   -0.0164   -0.2295   -0.1311
    0.1311   -0.1639    0.0492


>> A*iA % check

ans =
    1.0000         0         0
   -0.0000    1.0000    0.0000
    0.0000   -0.0000    1.0000
```