Name: Score: /100

This exam is closed-book.

- You must show ALL of your work for full credit.
  - Please read the questions carefully.
  - Please check your answers carefully.
- Calculators may NOT be used.
  - Please leave fractions as fractions, but simplify them, etc.
  - I do not want the decimal equivalents.
- Cell phones and other electronic communication devices must be turned off and stowed along with your backpack at the front to f the room.
- Please do not write on the backs of the exam or additional pages.
  - The instructor will grade only one side of each page.
  - Extra paper is available from the instructor.
- Please write your name on every page that you would like graded.

1	2	3	4	5	6

1. (20 points) Consider the n-link rigid robot dynamic equation

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + B\dot{q} + g(q) = u.$$

Identify by name and formula each variable in the above dynamic equation.

2. (20 points) Consider again the n-link rigid robot dynamic equation.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + g(q) = u.$$

You wish to design a joint space inverse dynamics controller. Write the expressions for the inner loop control u and the outer loop control a_q.

3. (25 points) Consider the two-link planar system consisting of a revolute joint rotating about a pivot mounted on a horizontal track. Derive the dynamic equations for this system.

4. (10 points) Define skew symmetry and show that

$$N(q, \dot{q}) = \dot{D}(q) - 2C(q, \dot{q})$$

is skew symmetric.

5. (20 points) Show that the two-link revolute joint arm with remotely driven link is linear in parameters. The manipulator has the kinetic energy

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

where

$$D(q) = \begin{bmatrix} m_1\ell_{c_1}^2 + m_2\ell_1^2 + I_1 & m_2\ell_1\ell_{c_2}\cos(q_2 - q_1) \\ m_2\ell_1\ell_{c_2}\cos(q_2 - q_1) & m_2\ell_{c_2}^2 + I_2 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & m_2\ell_1\ell_{c_2}\sin(q_2 - q_1) \\ -m_2\ell_1\ell_{c_2}\sin(q_2 - q_1) & 0 \end{bmatrix},$$

and

$$P = m_1 g\ell_{c_1}\sin q_1 + m_2 g\left(\ell_1\sin q_1 + \ell_{c_2}\sin(q_1 + q_2)\right)$$

Find Θ and $Y(q, \dot{q}, \ddot{q})$.

6. (5 points) State the Euler-Lagrange equation for an n-DOF system, and define all of the variables in it, giving formulas where appropriate.

Formula Sheet

A set of **Basic Homogeneous Transformations** that generate $SE(3)$

$$\mathrm{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{y,\beta} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{z,\gamma} = \begin{bmatrix} c_\gamma & -s_\gamma & 0 & 0 \\ s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the **Denavit-Hartenberg (DH) convention**, A_i is the product of four basic transformations,

$$\begin{aligned} A_i &= \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} \\ &= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$