Name: Score: /100

This exam is closed-book.

- You must show ALL of your work for full credit.
  - Please read the questions carefully.
  - Please check your answers carefully.
- Calculators may NOT be used.
  - Please leave fractions as fractions, but simplify them, etc.
  - I do not want the decimal equivalents.
- Cell phones and other electronic communication devices must be turned off and stowed along with your backpack at the front to f the room.
- Please do not write on the backs of the exam or additional pages.
  - The instructor will grade only one side of each page.
  - Extra paper is available from the instructor.
- Please write your name on every page that you would like graded.

1	2	3	4	5	6	7	8

1. (15 points) Find $\dot{p}^0$, the linear velocity with respect to the base frame, of a point $p^1 = \begin{bmatrix} 0 & 2 & 0 \end{bmatrix}^T$ attached to a moving frame F_1, if the moving frame is rotating at angular velocity $\omega = \hat{k}$ rad/s and translating along the x_0 axis at a rate of 1 m/s.

2. (25 points) Consider the manipulator whose DH parameters are given in the table below. Compute the geometric Jacobian.

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	a_2	0	d_2^*	0
3	0	0	0	θ_3^*
4	a_4	0	d_4^*	0

3. (5 points) Determine, based on the expression for the Jacobian that you found in the previous problem, whether singularities will occur. It is sufficient to give an equation in the appropriate variables (and only the appropriate variables). Solving for them is not necessary.

4. (10 points) Design a cubic trajectory that goes from $q(t_0) = 1$ to $q(t_1) = 3$ with $\ddot{q}(t_0) = 0$, and $\dot{q}(t_0) = 0$, where $t_0 = 0$ and $t_1 = 1$ by constructing and solving the appropriate matrix equation.

5. (25 points) Consider a spherical wrist consisting of three links. If the first rotates about x_0 having $\theta_1(t) = 5t$, the second rotates about y_1 with angle $\theta_2(t) = 3t$, and the third rotates with angle $\theta_3(t) = 2t$ about the fixed z_0 axis, what is the angular velocity of the third with respective to the fixed frame, expressed in the coordinates of the fixed frame. (Hint: Don't just try to plug this into a memorized formula. Either derive the correct formula or take care to find the correct rotation matrices and angular velocities.)

6. (5 points) A robotic vacuum of radius 2 is centered at location $(0, 0)$ in a room. It must avoid a triangular object with corners at $(5, -1)$, $(4, 3)$, and $(3, 1)$. Find the shortest distance to each side of the triangular object.

7. (5 points) Give the formula for the workspace repulsive force that should be used to keep the robot from hitting the triangular object. If $\eta_i = 1, \forall i$ and $\rho_0 = 1$, find the repulsive force corresponding to the closest point on the triangular object. You need not calculate a final value, but should supply all values required by the formula.

8. (10 points) State the Euler-Lagrange equation for an n-DOF system, and define all of the variables in it, giving formulas where appropriate.

Formula Sheet

A set of **Basic Homogeneous Transformations** that generate $SE(3)$

$$\text{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Rot}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Rot}_{y,\beta} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Trans}_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Rot}_{z,\gamma} = \begin{bmatrix} c_\gamma & -s_\gamma & 0 & 0 \\ s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the **Denavit-Hartenberg (DH) convention**, A_i is the product of four basic transformations,

$$\begin{aligned} A_i &= \text{Rot}_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}\text{Rot}_{x,\alpha_i} \\ &= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$