This exam is closed-book.

- You must show ALL of your work for full credit.
  - Please read the questions carefully.
  - Please check your answers carefully.

- Calculators may NOT be used.
  - Please leave fractions as fractions, but simplify them, etc.
  - I do not want the decimal equivalents.

- Cell phones and other electronic communication devices must be turned off and stowed under your desk.

- Please do not write on the backs of the exam or additional pages.
  - The instructor will grade only one side of each page.
  - Extra paper is available from the instructor.

- Please write your name on every page that you would like graded.
1. (10 points) Find the $H$ matrix corresponding to the following transformation: First rotate by $\pi/2$ about the fixed $y$-axis. Call the new frame $F_1$. Then translate 7 cm along the current $y$-axis. Call the new frame $F_2$. Then rotate by $\pi/4$ about the current $z$-axis. Call the new frame $F_3$. Finally translate 5 cm along the fixed $z$-axis. Call this new frame $F_4$. Please first evaluate the expression symbolically (i.e. in terms of variables) and then evaluate the resulting expression by substituting the values of the variables.
2. (5 points) If joint $J_i$ is positioned at joint angle $\pi/4$, and link $L_i$ has link length 110 mm, and link twist $\pi/2$, what is $A_i$?
3. (5 points) Identify the DH parameters for the matrix

\[ A_i = \begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 8 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}. \]

Assume that all angle measurements are between 0 and 2\(\pi\).

\[ a_i = \]
\[ \alpha_i = \]
\[ d_i = \]
\[ \theta_i = \]
4. (5 points) Give the expression for $T_5^3$ in terms of the appropriate $A_i$. 

5. (5 points) Give the expression for $T_3^5$ in terms of the appropriate $A_i$. 
6. (10 points) Using the $A_i$ and the $H$ you just found in problem (1), determine the following. If $p^2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, what are the coordinates of $p$ with respect to the fixed frame?
7. (5 points) Forward kinematics determines the position and orientation of the end effector from which of the following: (Circle all that apply.)

forces on links

joint angles

joint torques

link geometry

link moment of inertia

8. (5 points) What is the definition of a skew symmetric matrix?
9. (10 points) Please select frames of reference $F_0$, $F_1$, and $F_2$ and generate the table of DH parameters. Please place $F_0$ on the solid surface below the cylinder. If you find you need variables not listed in the diagram, please define and use them.

Figure 1.15: The Seiko RT3300 Robot cylindrical robot. Cylindrical robots are often used in materials transfer tasks. (Photo courtesy of Epson Robots.)
10. (5 points) Starting from Frame $F_0$, rotate first by $\theta_1$ about the fixed $x$-axis and then by $\theta_2$ about the current $z$-axis to orient Frame $F_1$. Find the rotation matrix representing this transformation.

11. (5 points) Starting from Frame $F_0$, rotate first by $\theta_1$ about the fixed $x$-axis and then by $\theta_2$ about the fixed $z$-axis to orient Frame $F_1$. Find the rotation matrix representing this transformation.
12. (10 points) A homogeneous transformation $H$ describes a rotation $R$ and a translation $d$.

(a) Give the expression for $H$ in terms of $R$ and $d$.

(b) Give the expression for $H^{-1}$ in terms of $R$ and $d$. 
13. (5 points) Determine the singularities of the system whose Jacobian is
\[ J_{11} = \begin{bmatrix}
-c_1 d_3 & 0 & -s_1 \\
-s_1 d_3 & 0 & c_1 \\
0 & 1 & 0
\end{bmatrix}. \]

14. (5 points) What is the origin of frame $F_1$ with respect to frame $F_0$ if
\[ A_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 6 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 5 \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}. \]
15. (10 points) Where \( J_v = [J_{v_1} J_{v_2} \ldots J_{v_n}] \) and \( J_\omega = [J_{\omega_1} J_{\omega_2} \ldots J_{\omega_n}] \), give the expressions for \( J_{v_i} \) and \( J_{\omega_i} \).
Formula Sheet

A set of Basic Homogeneous Transformations that generate $SE(3)$

$$
\text{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Rot}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\text{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Rot}_{y,\beta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\beta & 0 & s_\beta \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\text{Trans}_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Rot}_{z,\gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\gamma & -s_\gamma & 0 \\ s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

In the Denavit-Hartenberg (DH) convention, $A_i$ is the product of four basic transformations,

$$
A_i = \text{Rot}_{z,\theta_i} \text{Trans}_{z,d_i} \text{Trans}_{x,a_i} \text{Rot}_{x,\alpha_i}
$$

$$
= \begin{bmatrix} c_\theta_i & -s_\theta_i & 0 & 0 \\ s_\theta_i & c_\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_\theta_i & -s_\theta_i & c_{\alpha_i} & s_{\theta_i} \alpha_i & a_i c_{\theta_i} \\ s_\theta_i & c_\theta_i & c_{\alpha_i} & -s_{\theta_i} \alpha_i & a_i c_{\theta_i} \\ 0 & 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & s_{\alpha_i} & 0 & c_{\alpha_i} & d_i \end{bmatrix}
$$