Control of Single Switch Inverters

Shweta Hegde, *Student Member, IEEE,* Afshin Izadian, *Senior Member, IEEE*

***Abstract*— This paper presents a control approach for a single switch inverter. The inverter exhibits non-minimum phase behavior in some of the operating modes. A dual feedforward controller was employed with an adaptive PI controller to obtain perfect tracking performance. The controller successfully isolates the non-minimum phase part of the system from the minimum phase. Both separated minimum phase and non-minimum phase sub-systems were used in the dual feedforward scheme to generate desired references. The non-minimum phase dynamics are transformed to minimum phase by using an inverse system transfer function as parallel compensator. The adaptive PI controller was used to track the voltage references.**

I. Introduction

A new inverter shown in Fig. 1, was introduced to generate a pure sinusoidal output voltage [1]. The system behaves like a non-minimum phase system in all operating ranges except when it is operating to produce negative half cycle in buck mode. When addressed from a control perspective, the right half plane zeros or the non-minimum phase zeros in the transfer function complicate the control design scheme [3],[4]. The response of such a system is characterized by undershoots and overshoots [2].

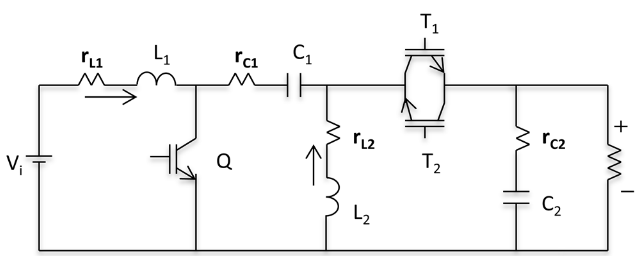

Fig. 1. Schematic diagram of the new Inverter

These systems can be controlled if they could be converted to minimum phase systems. Parallel feedforward compensators can be used to convert any plant into a minimum phase system. Parallel compensators have been successfully implemented in [2], [5]-[9] and are proven to be a more efficient way of controlling non-minimum phase systems compared to pole-zero cancellation techniques [10]-[11]. For non-minimum phase systems, pole-zero cancellation can lead to having unstable structure of feedback controller. However, this method uses a compensator T(s) which is not a part of the plant but is derived to make the plant a minimum phase system.

This paper uses a Dual Feed forward Predictive Control structure to solve the tracking problem of a non-minimum phase system.

In general the DFPC may provide perfect tracking for [12], Stable and unstable systems, Biproper and strictly proper systems, Minimum and non-minimum phase systems.

In this case, the feedforward controller is used to provide either the feedforward prediction or the ballistic response. The feedforward controllers are based on the plant model [12]. The feedback controller is responsible for tracking reference signals. For perfect tracking, the reference signal is divided into two signals namely a reference signal that can be inverted by the ballistic response and a prediction of the path that the plant output will follow based on the ballistic response. The feedback controller is designed to result in perfect tracking performance. A simple PI controller can be used for regulations. However, for reference tracking a gain adaptation is utilized to constantly tune the gains of the controller [16]-[19].

Section II covers the method of dual feedforward control and Section III covers the application of this control technique to the system. Section IV presents the simulation results.

II. Dual Feedforward Predictive Control

This method of feedforward control is used to force the non-minimum phase system to behave like a minimum phase system. In this method, the plant is split into two parts to generate two signals. One signal is to make the plant track $r_{ff}(t)$ with a feedforward control signal $u_{ff}(t)$ that drives the plant to track the reference signal. The signals produced by the feedforward transfer functions are assumed to contain bounded energy and have no influence on the closed loop stability [12]. For perfect tracking, the error should reach zero which can be accomplished using various types of controller including a simple gain [2]. However, in the new inverter circuit, an adaptive PI controller is required to adjust the gains continuously. The block diagram, for the structure of a dual feedforward predictive control (DFPC) is shown in Figure 2.

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S. Hegde and A. Izadian are with the Energy Systems and Power Electronics Laboratory at the Purdue School of Engineering and Technology, IUPUI. Email: aizadian@iupui.edu.

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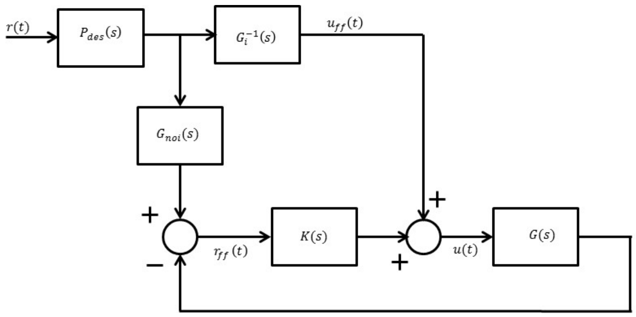

Fig. 2. Block Diagram of Dual Feedforward Predictive Control

The stable and causal blocks $G_{noi}(s)$ and $G_i^{-1}(s)$ are the non–invertible and invertible parts of the plant. The non-invertible part refers to the non-minimum phase and unstable part of the plant where as the invertible part refers to the minimum phase and stable part of the plant. $P_{des}(s)$ is the design parameter that determines the reference signal and the feedforward control signal. The conditions to be satisfied by the design parameter are:

1. The steady state gain from $r(t)$ to $r_{ff}(t)$ must be unity gain i.e. $P_{des}(0)G_{noi}(0) = 1$.
2. The feedforward transfer functions $FF1 = P_{des}(s)G_{noi}(s)$ and $FF2 = P_{des}(s)G_i^{-1}(s)$ must be proper.

The first condition is required so that the steady state reference equals the actual reference; $r_{ff}(t) = r(t)$ .The second condition is required to make the feedforward controller realizable from the hardware point of view. The conclusion drawn from the second condition is that $P_{des}(s)$is stable and the relative degree of $P_{des}(s)$ is greater than or equal to the relative degree of $G_i(s)$.

Also, the controller $K(s)$ should be designed to guarantee internal stability. The nominal tracking requirements are satisfied by the feedforward paths and the feedback controller focuses on correcting model inaccuracies and disturbance rejection.

In particular, $P_{des}(s)G_{noi}(s)$ determines the class of signal that has to be perfectly tracked and $P_{des}(s)G_i^{-1}(s)$ determines the associated feedforward control signal to achieve perfect tracking. Consider the plant to have a transfer function as follows:

$$G(s) = \frac{K_{DC}N_{mp}(s)N_{nmp}(s)}{D_s(s)D_u(s)}$$

where K_{DC} is the DC gain of the system, $N_{mp}(s)$ is the minimum phase polynomial of the numerator while $N_{nmp}(s)$ is the non-minimum phase polynomial of the numerator. $D_s(s)$ is the stable denominator polynomial and $D_u(s)$ is the unstable denominator polynomial.

The transfer function is decomposed into two parts: $G_i(s)$ and $G_{noi}(s)$. $G_i(s)$ contains the minimum phase numerator polynomial and the denominator polynomial and $G_{noi}(s)$ contains the non-minimum phase numerator polynomial.

$$G_i(s) = \frac{K_{DC}N_{mp}(s)}{D_s(s)D_u(s)} \quad \text{and} \quad G_{noi}(s) = N_{nmp}(s)$$

It is to be noted that the transfer functions $G_i(s)$ and $G_{noi}(s)$ are not proper and cannot be realized as individual systems. This leads to the selection of the design parameter $P_{des}(s)$ such that the feedforward transfer functions FF1 and FF2 are proper and realizable.

The control effort is a sum of the feedforward control signal obtained at the output of the block ${G_i}^{-1}(s)$ and the feedback control signal obtained at the output of $K(s)$.

III. Feedback Controller

The feedback controller needs to be designed to provide zero tracking error. In some cases, a simple gain or simple PI controller can be very effective [2]. However, PI controllers give best results when the goal of control is regulation. Also, using a simple PI controller needs tuning of the gains offline. In our case, the signal to be tracked is continuously varying (sine wave) and thus an adaptive PI controller structure was considered suitable. Also, the gains of adaptive controller are tuned automatically online [6]. Any change in the control objectives or change in the plant parameters can be compensated by using online tuning of the gains of the controller [14].

The self-tuning PI controller is viewed as a non-linear controller as the gains K_p and K_i are varying continuously. It is not necessary for the gains to converge to a constant value.

The equations for the proportional gain K_p and integral gain K_i are obtained from [14] as:

$$\begin{cases} \dot{K}_p = -\gamma e y_1 \\ \dot{K}_i = -\gamma e y_2 \end{cases} \quad (1)$$

where $\gamma > 0$ is the adaption gain, e is the error between the plant output and the reference input, y_1 is the output of the proportional block and y_2 is the output of the integral block of the controller.

IV. Controller Design for New Inverter

A. Buck Operation

When the inverter operates in buck mode to produce positive cycle of output voltage, the control to output transfer function has non-minimum phase behavior.

For buck operation with duty cycle as 30%, and the following parameters:$L_1 = 1\mu H$, $L_2 = 25\mu H$, $C_1 = 1\mu F$, $C_2 = 75mF$ and $R = 5$, the control to output transfer function is determined as follows:

$$G_d^v(s) = \frac{(-s+5.58)(s^3+0.4775s^2+3.5821s+0.1897)}{s^4+0.0398s^3+49.36s^2+0.1283s+0.0026}$$

The transfer function is decomposed as follows:

$$G_{mp}(s) = \frac{(s^3 + 0.4775s^2 + 3.5821s + 0.1897)}{s^4 + 0.0398s^3 + 49.36s^2 + 0.1283s + 0.0026}$$

$$G_{nmp}(s) = (-s + 5.58)$$

The minimum phase part is the stably and causally invertible G_i and the non-minimum phase part is considered to be causally non-invertibleG_{noi}.

$$G_i^{-1}(s) = \frac{s^4 + 0.0398s^3 + 49.36s^2 + 0.1283s + 0.0026}{(s^3 + 0.4775s^2 + 3.5821s + 0.1897)}$$

$$G_{noi}(s) = (-s + 5.58)$$

Following the design requirements for the choice of $P_{des}(s)$ the relative degree of $P_{des}(s)$ is chosen to be equal to 1 and has the structure as:

$P_{des}(s) = \frac{k}{\alpha s+1}$ where α is varied to determine a suitable response.

In addition, where $G_{noi}(0) = 5.58$ yields $P_{des}(0) = k = 5.58^{-1}$

Thus, $P_{des}(s) = \frac{5.58^{-1}}{\alpha s+1}$

The presence of right half plane (RHP) zero means that the step response will have an undershoot that is related to the value of α. The value of α is determined such that the undershoot is reduced and the response is fast. It is a trade-off between the undershoot and the response time. For smaller values of α, the response is fast but the undershoot is larger and for larger values of α, the undershoot is less but the response is slower.[11]. The step response of this system is shown in Fig. 3 for various values of $\alpha = 5$ is chosen to give the best result.

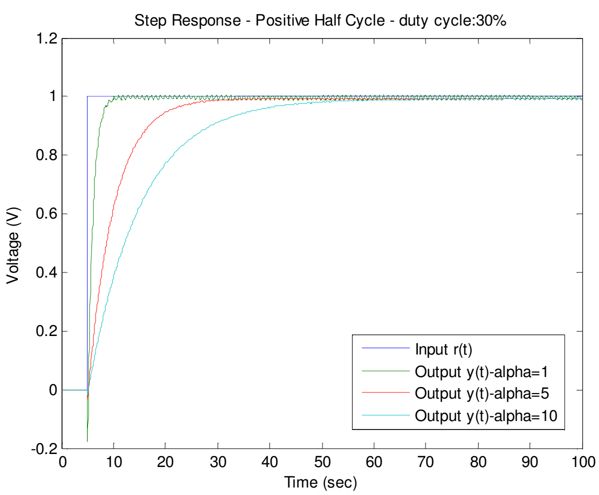

Fig. 3. Step Response of system for positive half cycle for duty cycle of 30%

When the inverter operates in buck mode to produce negative cycle of output voltage, the control to output transfer function has minimum phase behavior.

For operation with duty cycle as 30%, the control to output transfer function is determined as follows:

$$G_d^v(s) = \frac{s^4 + 0.1276s^3 + 49.24s^2 + 0.6402s + 0.0297}{s^4 + 0.0398s^3 + 49.36s^2 + 0.1283s + 0.0026}$$

The transfer function is decomposed as follows:

$$G_{mp}(s) = \frac{s^4 + 0.1276s^3 + 49.24s^2 + 0.6402s + 0.0297}{s^4 + 0.0398s^3 + 49.36s^2 + 0.1283s + 0.0026}$$

$$G_{nmp}(s) = 1$$

The minimum phase part is the stably and causally invertible G_i and the non-minimum phase part is considered to be causally non-invertibleG_{noi}.

$$G_i^{-1}(s) = \frac{s^4 + 0.0398s^3 + 49.36s^2 + 0.1283s + 0.0026}{s^4 + 0.1276s^3 + 49.24s^2 + 0.6402s + 0.0297}$$

$G_{noi}(s) = 1$.

The relative degree of $P_{des}(s)$ is required to be greater than or equal to the relative degree of $G_i(s)$. Since the relative degree of $G_i(s)$ is zero, the relative degree of $P_{des}(s)$ is assumed to be zero.

$$P_{des}(s) = \frac{(s+k)}{\alpha s + 1}$$

In addition, $G_{noi}(0) = 1$ and thus $P_{des}(0) = k = 1$, Therefore,

$$P_{des}(s) = \frac{s+1}{\alpha s + 1}$$

The value of α is determined from the step response of the system operating to produce negative peak as shown in Fig. 4. The value of α was not considered to be one because then it would reduce the design parameter $P_{des}(s)$ to one. In this case, there is no undershoot as the system has minimal phase behavior. The value of α was selected to be 5 to be in coherence with $P_{des}(s)$ of other case.

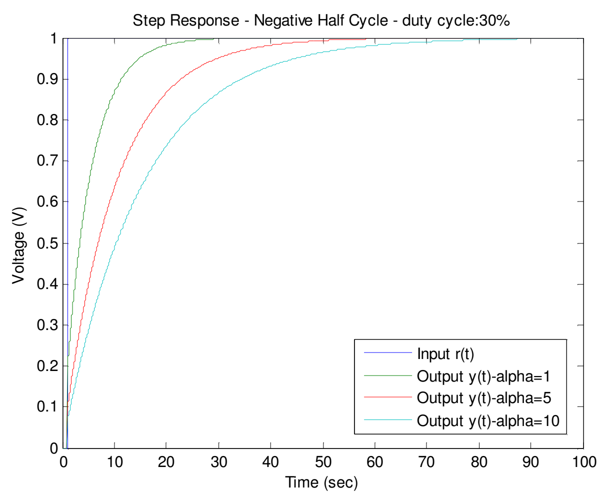

Fig. 4. Step Response of system for negative half cycle for duty cycle of 30%

The system when operating to produce a complete cycle for duty cycle of 30% was simulated using the DFPC control and adaptive PI. Fig. 5 shows the tracking of the plant output. The plant output follows the reference signal exactly at every instant of time.

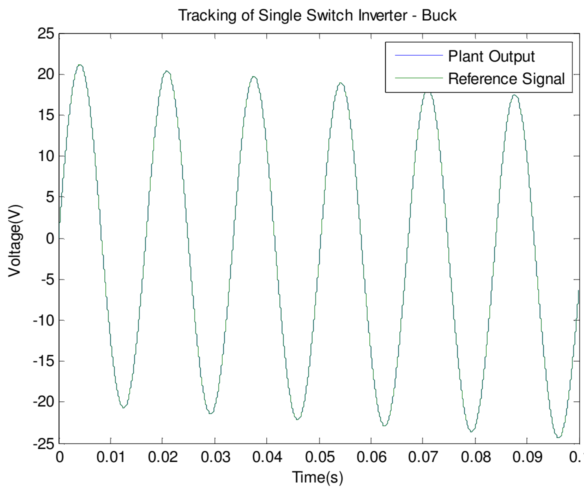

Fig. 5. Perfect tracking for inverter operation with duty of 30%

The output of the plant was made to track the reference signal exactly.

B. Boost Operation

When the inverter operates in boost mode to produce positive cycle of output voltage, the control to output transfer function has non-minimum phase behavior.

For boost operation with duty cycle as 70%, and the following parameters: $L_1 = 1\mu H$, $L_2 = 25\mu H$, $C_1 = 1\mu F$, $C_2 = 75mF$ and $R = 5$, the control to output transfer function is determined as follows:

$$G_d^v(s) = \frac{(-s + 1.1681)(s^3 + 0.2911s^2 + 3.089s + 0.1640)}{s^4 + 0.0236s^3 + 10.96s^2 + 0.0701s + 0.0004}$$

The transfer function is decomposed as follows:

$$G_{mp}(s) = \frac{(s^3 + 0.2911s^2 + 3.089s + 0.1640)}{s^4 + 0.0236s^3 + 10.96s^2 + 0.0701s + 0.0004}$$

$$G_{nmp}(s) = (-s + 1.1681)$$

The minimum phase part is the stably and causally invertible G_i and the non-minimum phase part is considered to be causally non-invertible G_{noi}.

$$G_i^{-1}(s) = \frac{s^4 + 0.0236s^3 + 10.96s^2 + 0.0701s + 0.0004}{(s^3 + 0.2911s^2 + 3.089s + 0.1640)}$$

$$G_{noi}(s) = (-s + 1.1681)$$

The design parameter $P_{des}(s)$ is found to have the structure as

$$P_{des}(s) = \frac{k}{\alpha s + 1}$$

In addition, $G_{noi}(0) = 1.1681$ and thus $P_{des}(0) = k = 1.1681^{-1}$

Therefore,

$$P_{des}(s) = \frac{1.1681^{-1}}{\alpha s + 1}$$

The value of α is determined from the step response of the system. The trade-off leads to selection of $\alpha = 5$. The step response of the system is shown in Fig. 6.

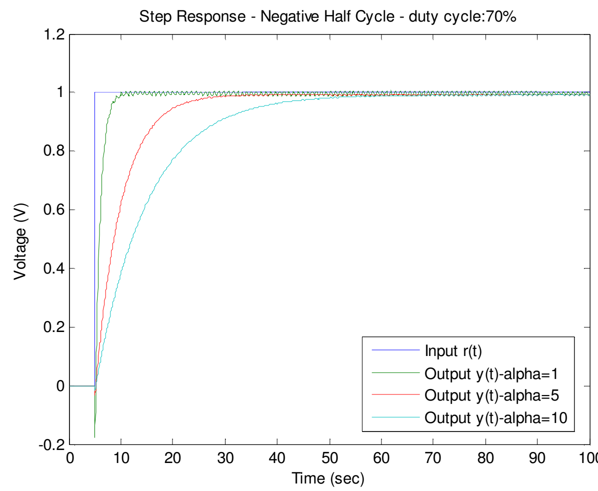

Fig. 6. Step Response of system for positive half cycle for duty cycle of 70%

From Fig 6., it can be seen that for value α=1 , the undershoot is large while the time taken for the response to settle is very small. For $\alpha = 10$, the undershoot is small but the response time is very large. The best response can be obtained for $\alpha = 5$.

When the inverter operates in boost mode to produce negative cycle of output voltage, the control to output transfer function has non-minimum phase behavior.

For operation with duty cycle as 70%, the control to output transfer function is determined as follows:

$$G_d^v(s) = \frac{(-s + 0.0009)(s^3 + 0.0183s^2 + 10.93s + 0.0689)}{s^4 + 0.0236s^3 + 10.96s^2 + 0.0701s + 0.0004}$$

The transfer function is decomposed as follows:

$$G_{mp}(s) = \frac{(s^3 + 0.0183s^2 + 10.93s + 0.0689)}{s^4 + 0.0236s^3 + 10.96s^2 + 0.0701s + 0.0004}$$

$$G_{nmp}(s) = (-s + 0.0009)$$

The minimum phase part is the stably and causally invertible G_i and the non-minimum phase part is considered to be causally non-invertible G_{noi}.

$$G_i^{-1}(s) = \frac{s^4+0.0236s^3+10.96s^2+0.0701s+0.0004}{(s^3+0.0183s^2+10.93s+0.0689)},$$

$$G_{noi}(s) = (-s + 0.0009).$$

The structure of $P_{des}(s)$ is selected to be as follows:

$$P_{des}(s) = \frac{k}{\alpha s + 1}$$

Considering, $G_{noi}(0) = 0.0009$ and thus $P_{des}(0) = k = 0.0009^{-1}$

Therefore,

$$P_{des}(s) = \frac{0.0009^{-1}}{\alpha s + 1}$$

The value of α is determined from the step response of the system as shown in Fig. 7. The value of α which gives the best trade-off is selected. The undershoot is unacceptable for α=1 while the time taken for the response to settle is unacceptable for α=10.

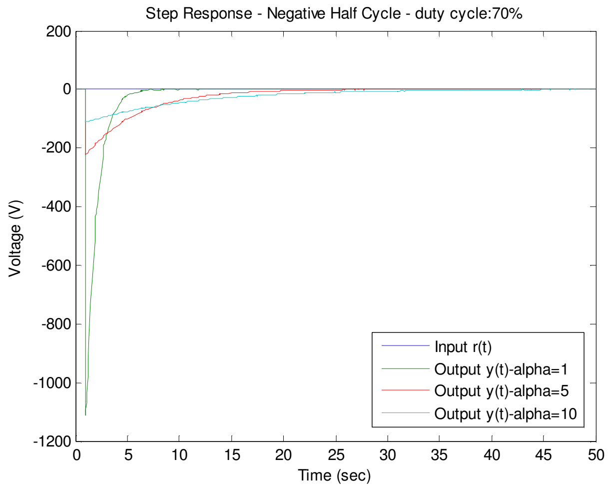

Fig. 7. Step Response of system for negative half cycle for duty cycle of 70%

The simulation result for perfect tracking of this system when operating to boost the input voltage with duty cycle of 70% is shown in Fig. 8.

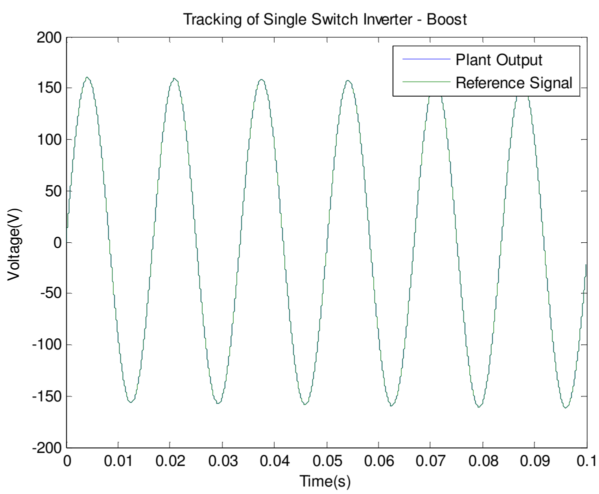

Fig. 8. Perfect tracking for inverter operation with duty of 70%

A single structure of feedback controller was designed for the inverter operating to produce positive and negative half cycles. An adaptive PI controller was selected and designed using MATLAB/SIMULINK. The gains of the controller were taken from equation (1).

V. Conclusion

A controller was designed for perfect tracking of a new single switch inverter which behaves like a non-minimum phase system when operating in certain operating ranges. A dual feedforward predictive control technique was used which converted the non-minimum phase system to a minimum phase system by using a parallel compensator. The plant was decomposed into two parts which produce two different signals: one of which was the reference that was tracked by the plant and the other was the feedforward control signal. A feedback controller was used to eliminate the tracking error. The sum of the signal from the feedback controller and the feedforward control signal was the input to the plant which tracks a reference signal perfectly. An adaptive PI controller was chosen over fixed gain PI so that the gains of the controller could be tuned real-time to compensate for changes in plant behavior or control objective. The controller was simulated using MATLAB/SIMULINK and resulted in perfect tracking performance.

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