# A Model Linearization Technique for Hydraulic Wind Power Systems

Masoud Vaezi, Afshin Izadian, *Senior Member, IEEE,* Majid Deldar
*Energy Systems and Power Electronics Laboratory*
*Purdue School of Engineering and Technology, IUPUI*
*Indianapolis, IN, USA*
aizadian@iupui.edu

***Abstract*-Hydraulic wind turbines are highly nonlinear and work severely under variable disturbances such as load on the generators and the wind speed. For further analysis control implementation on these types of systems a suitable linearized model is needed. Due to the disturbances, the system has a wide range of operating points. Therefore, linearized models on different operating regimes will be needed. Finding the best operating points, to linearize the nonlinear system at, can improve the accuracy of the linearization as well as the stability of the system. This paper propose a new automatic algorithm to indicate the suitable operating points and linearized models.**

## I. Introduction

Conventionally, wind turbines utilize gearboxes to transmit low speed high torque power from blades to the generator. The cost of gearbox can be up to 34% of wind turbine. It needs several overhauls and may be replaced several times in a 20 year lifespan of a wind turbine. Therefore, alternative replacements can be used to transfer the energy in form of pressurized fluid such as Hydrostatic Transmission. In this method, kinetic energy of the turbine is converted to hydrostatic pressurized fluid at the pump to transfer the energy to the generator on ground level.

To reach desired objectives from a hydrostatic transmission system for wind turbine application, the system needs to be controlled appropriately. The speed control of hydraulic wind power systems is challenging, since it is a nonlinear system under random disturbance inputs i.e. wind speed. The nonlinearities in such system are originated from nonlinear behavior of components such as check valves, directional valves and more importantly the proportional valve. These nonlinearities will cause behavioral changes and variations in the system. Therefore, the speed control of the system would require an in-depth modeling. The controller's performance depends on states variables while the system is influenced by large input variations in a wide operating range. Proper controllers can be designed using the linearized models.

Previous studies focused to identify the governing operation of hydrostatic transmission systems [1-9]. They accounted hydraulic parameters of pumps, motors and valves to develop a realistic model. The nonlinear systems with wide range of operation can be linearized to simplify the controller structure. Therefore, the model linearization approach is required to identify proper operating points and a technique to represent the original system's state variables.

This paper introduces a novel algorithm to identify the operating points at which the nonlinear hydraulic wind power system can be linearized and represented. An optimal approach reduces the mismatch errors in further implementation of multiple model linearized system compared to the original nonlinear system [10,22].

## II. Hydraulic Wind Power System Operation

The Hydrostatic Transmission System (HTS) comprises pump, motors, pressure control and flow control valves. The pump, coupled with the turbine, supplies flow to the hydraulic circuit. The flow is distributed through a proportional valve between the main and auxiliary motor. Main motor is connected to main generator and the auxiliary motor captures the excess energy that cannot be captured by the main motor. Safety components are also implemented in the hydraulic circuit to protect against high pressure and reverse flow directions. Check valves are needed to ensure fluid is only directed in the desired direction and pressure relief valves protect the circuit against sudden overpressure. Considering nonlinear dynamics of each hydraulic component, the governing equations of flow, speed and torque can be obtained. Fig.1 demonstrates schematics of the HTS used in this paper. In order to regulate the speed and consequently the generated power from the generator, a controlled proportional flow valve is required to distribute the hydraulic fluid delivery of the pump to the main hydraulic motor and the excess bypass flow to the hydraulic pump. Optimal speed control of hydraulic systems is represented in [7], [11], and [12].

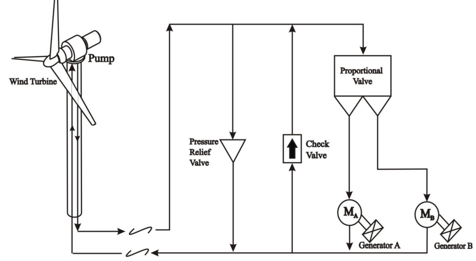


Fig. 1. The schematic of a single wind turbine system.

## III. Mathematical Modeling and Problem Definition

The state space representation of the hydraulic wind power system can be derived by considering the integrated

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configuration of the hydraulic components such as pumps, proportional valves and check valves. The nonlinear model of hydraulic circuit components [11], [13-17] and the nonlinear state space representation of the hydraulic wind energy transfer is introduced in [2,6,8,9]. Considering dynamics of each hydraulic component, governing equations of flow and torque are derived. Those equations are used to represent the HTS in form of nonlinear state space equations. The representation of the system model with energy storing state variables defined in vector $x$ can be obtained as follows [2]:

$$x = \begin{bmatrix} P_p & P_{mA} & P_{mB} & \omega_{mA} & \omega_{mB} \end{bmatrix}^T \tag{1}$$

where the state variables are pump pressure, the main motor inlet pressure, the auxiliary motor inlet pressure, main motor and auxiliary motor speeds, respectively. The input vector is defined as $U = \begin{bmatrix} \omega_p & h_i & T_{mA} \end{bmatrix}^T$ with the pump speed, valve position and the load torque on main motor (motor A), respectively. Nonlinear state space model of the system is represented as [18]:

$$\begin{cases} y = h(x) \\ \dot{x} = f(x) + g(x)U \end{cases} \tag{2}$$

where the functions $f(x)$ and $g(x)$ are defined as follows: [18]

$$f(x) = \begin{Bmatrix} \frac{\beta}{V_1}\left\{ -K_p P_p - \left( C_D A_{max} \sqrt{\frac{2(P_p - P_{mB})}{\rho}} \right) \right\} \\ \frac{\beta}{V_2}\left\{ -D_{mA}\omega_A - K_{mA} P_{mA} \right\} \\ \frac{\beta}{V_3}\left\{ \left( C_D A_{max} \sqrt{\frac{2(P_p - P_{mB})}{\rho}} \right) - D_{mB}\omega_B - K_{mB} P_{mB} \right\} \\ D_{mA} P_{mA} - B_{mA}\omega_{mA} - T_{lmA} \\ D_{mB} P_{mB} - B_{mB}\omega_{mB} - T_{lmB} \end{Bmatrix} \tag{3}$$

$$g(x) = \begin{bmatrix} D_p & \frac{\beta}{V_1}\left\{ -\left( C_D \frac{A_{max}}{h_{max}} \sqrt{\frac{2(P_p - P_{mA})}{\rho}} \right) + \left( C_D \frac{A_{max}}{h_{max}} \sqrt{\frac{2(P_p - P_{mB})}{\rho}} \right) \right\} & 0 \\ 0 & \frac{\beta}{V_2}\left( C_D \frac{A_{max}}{h_{max}} \sqrt{\frac{2(P_p - P_{mA})}{\rho}} \right) & 0 \\ 0 & \frac{\beta}{V_3}\left( C_D \frac{A_{max}}{h_{max}} \sqrt{\frac{2(P_p - P_{mB})}{\rho}} \right) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

As it can be seen from the equations, the model is highly nonlinear as a result of nonlinear components such as proportional valve. The proportional valve consists of one inlet and two outlet orifices and a spool which changes the flow passage area of the outlet orifices. The fluid enters the valve and based on the position of valve spool, the flow is distributed between two outlets; main and auxiliary. The spool is displaced by applied current to its coil. Governing equation of flow rate for each outlet is obtained in (5). The pressure differential across an orifice and the passage area determine the rate of flow. Flow rate passing an orifice is obtained as [2]:

$$Q = C_D A \sqrt{\frac{2\Delta P}{\rho}} \tag{5}$$

where $A$ is the orifice area and $\Delta P$ is the pressure differential across the orifice. $C_D$ and $\rho$ are the discharge coefficient and the fluid density, respectively. Since the generator runs under electric load at synchronous speed, the constant speed of the hydraulic motor coupled with the generator is required to be maintained. The rotational speed of motors are functions of their flows which is regulated by utilizing a proportional valve. As mentioned earlier, operation of such a valve imposes nonlinearities in the system dynamics.

For the purpose of system analysis or a desired state control, a well-developed linear model can be obtained and utilized. However, this requires that the linearized system represent the nonlinear behavior of the system with a limited error on a large domain [19]. Hydraulic wind power system are susceptible to intermittent wind speed and range of control commands (to proportional valve) and experience a constantly varying electric load on the generator. From (5) it can be seen that the pressure differential variation disturbs flow of orifices. Thus, for maintaining the flow rate specifically for the main motor, the proportional valve must adjust the spool displacement to compensate for this disturbance.

Another source of disturbance for the valve performance is the pump speed. Wind speed variation changes the pump speed since it is connected to the turbine. Consequently, the amount of flow enters to the valve inlet varies. The variation of inlet flow affects the outlet flow so that the valve adjusts the orifice to compensate for this disturbance.

Even if the valve maintains a constant motor flow, its speed can deviates from synchronous speed due to pressure changes at the motor inlet that is applied by variation of the load on the generators. The governing equation for motor flow and delivered toque to the load is as follows:

$$I_m \frac{d\omega_m}{dt} = D_m P_m - C_v D_m \mu \omega_m - T_l \tag{6}$$

$$Q_m = D_m \omega_m + K_{ms} P_m . \tag{7}$$

It can be observed from (6) that even at a constant flow, the motor speed deviates if the pressure changes. For any pump and motor, flow slippage occurred between high pressure and low pressure parts reduced the flow rate. This slippage is proportioned to pressure as shown in (7). These types of nonlinear systems with wide range of operating points are usually linearized using multiple linear models to represent the whole system [22]. The linearization technique used in this paper is to utilize a local linear model for desired operating points. Multiple linearized models are therefore developed to cover the entire operating conditions. Each model should satisfactorily describe the plant in a specific domain. This linearized plant will have an effective range of linearization, in which the system generates minimum deviation from the original plant. Out of this domain, the linearized plant's performance is reduced hence a new plant with shifted operating conditions is required. A shared region that falls into effective domain of several plants may be approximated by a weighting factor associated to each model involved.

A linear combination of all linearized models represents the nonlinear system. In this method, any linear estimation technique such as multiple model adaptive estimation (MMAE) may be used to combine the information contained in the local models into a global description of the plant. State estimation can be utilized to track the transitions on-line to provide a means to control the system [20], [21].

Number of models in MMAE highly affects the stability of the estimation as well as the size of calculations. This variable is often determined by the range of disturbances on the system. Hydraulic transmission system test-bed in Energy Systems and Power Electronics Laboratory includes wind speed from 200 to 600 rpm, and valve position from 0 to 0.5 inches. Through a trial and error, 6 linearized models were identified to represent the hydraulic wind power transfer system. In the next section, an automatic method is discussed to determine proper operating points that result in minimum number of models with maximum accuracy.

## IV. Proposed Algorithm and Results

The algorithm is to determine a number of possible operating points and select some of them as suitable candidates. The primary goal is to minimize the mismatch of the nonlinear system with MMAE on all possible operating points. When nonlinear model is linearized on possible operating points, the linearized model is expected to represent the dynamics of a neighborhood from the operating point.

Existence of measurement and system noise cause a deviation from the operating point such that the linearized model starts to move away from nonlinear model dynamic. Hence for possible operating points further away from the point of linearization, the error is increased and may exceed from desired performance (7% in this paper). Therefore, a new model is required.

Then each model will have a designated area that covers part of the domain (possible operating points). Six models that cover most areas are selected to present the nonlinear system dynamics. The nonlinear model domain operating points might be covered by several linearized models (overlap) and might not be covered by the linearized models. To identify these models a weighing mechanism is proposed.

In order to weigh each model, every point in the operating surface must be assigned a value. The value of each point is determined from its likelihood to be covered by a nearby model. The more models cover a point the less worthy the point becomes. Equation (8) weighs each operating point on domain of linearized models. P_value is the assigned point value and NM is the number of models that cover the point. Knowing the total number of operating points in domain of each model, one can set the scaling factors C and K to generate enough resolution in recognizing the operating points, as follows:

$$P\_Value = K \times \frac{1}{(NM)^C} \quad . \tag{8}$$

By collecting the mapped weights of operating point in the domain of all linearized model, the models can be ranked. The model with highest weight is selected as the first candidate for linearized model. The weighs assigned to the operating points of the selected model are nullified and the model weighing and ranking is repeated to select the second, third and other linearized models. Fig. 2 illustrates the model selection procedure.

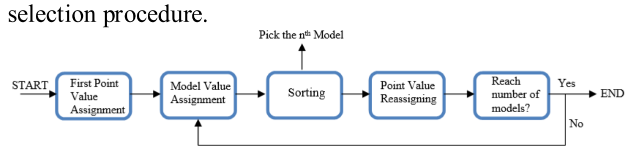


Fig. 2. Block diagram of the proposed algorithm.

For the first step, the value of operating points was manually determined with an even distribution in operating surface and a reasonable coverage by trial and error. Dividing this region into 6 models, a limited error was achieved where at maximum deviation reached 7%.

Using these models, different operating points of hydraulic wind power transfer system and selected operating point can be specified. It can be seen that some operating points are covered by overlap of several models and some operating points might not be covered. Fig. 3 demonstrates the maximum deviation of the nonlinear model outputs with that of the 6 linearized models.

Utilizing the specified models, Kalman filters were used in MMAE scheme to model the effect of measurement and system noise in the linearized models [10][22] and the algorithm was implemented with MATLAB/Simulink. The estimations of MMAE for the pressures are compared with the exact values from the nonlinear model of system in all possible operating points specified by control and disturbance input (valve position and wind speed respectively) to study the accuracy of implemented structure.

Considering a 7% maximum error between the exact values from the nonlinear system and MMAE, Fig. 4 demonstrates 80.75% coverage of the operating points by the adaptive estimation. Fig. 5 shows that the average of error between nonlinear states and MMAE estimated states is below 4%.

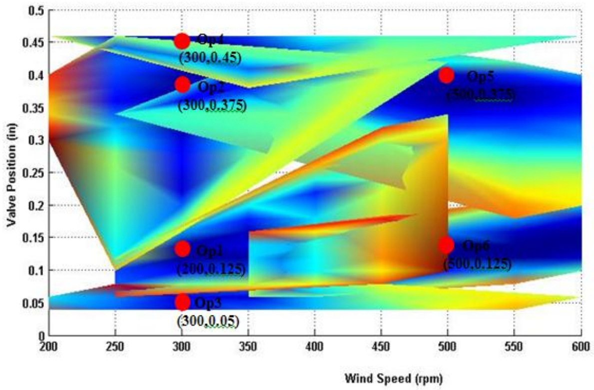


Fig. 3. Acceptable area of each model to describe the nonlinear system.

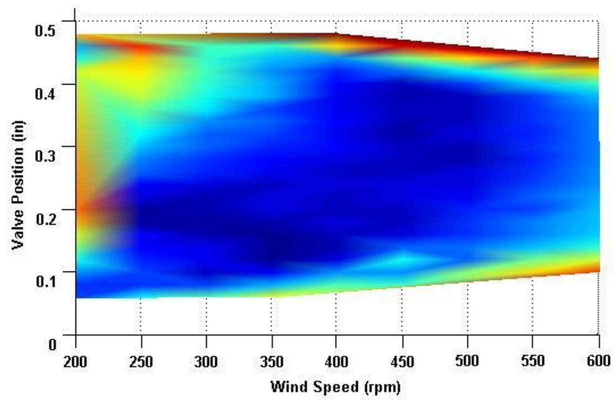


Fig. 4. Coverage of the MMAE over the whole operating regime.

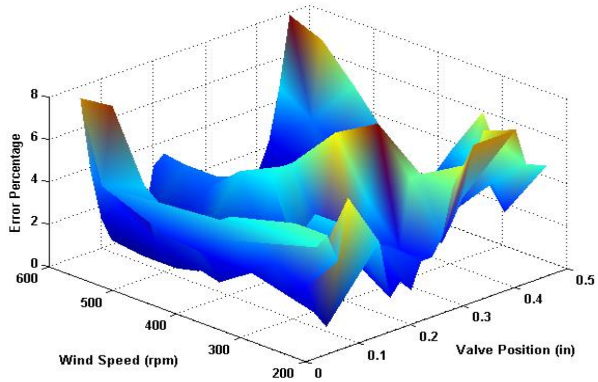


Fig. 5. Average of error between nonlinear states & MMAE estimated states

For the second simulation, using the proposed algorithm, Fig. 6 shows the optimized operating points and their corresponding model coverage. Considering the same criterion as 7% maximum error between the exact values from the nonlinear system and that of MMAE, Fig. 7 illustrates 86.2% coverage of the operating points by the adaptive estimation.

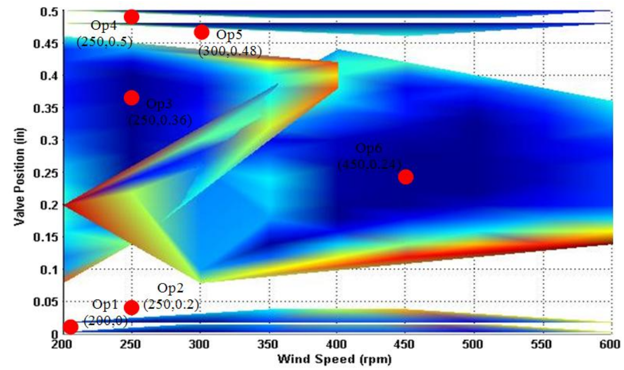


Fig. 6. Acceptable area of each model to describe the nonlinear system derived by proposed algorithm.

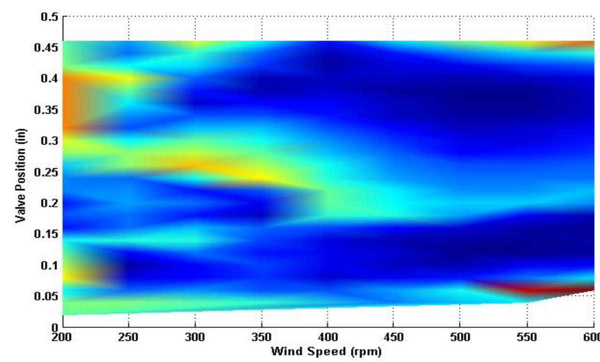


Fig. 7. Coverage of the MMAE over the whole operating regime achieved by proposed algorithm.

As it can be seen by models selected from the proposed algorithm, the overall accuracy of the MMAE has been increased. In addition, by comparing Fig. 5 and Fig. 8, it can be concluded that besides the increase in overall coverage of the operating regimes, the error between nonlinear system and the multiple model adaptive estimation was reduced. This improved the accuracy of the state estimation in the context of multiple model approach.

## V. Conclusion

Finding suitable operating points to linearize a hydraulic wind power transfer technology was accomplished using a model weighing and ranking mechanism. Properly selected operating points formed linearized models and used in their domain selection. A coverage of 86.2% was obtained using the proposed algorithm.

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